ACQUISITION OF FRACTION CONCEPTS ONE-HALF AND ONE-FOURTH BY FIRST-GRADE STUDENTS USING A CONSTRUCTIVIST APPROACH

By

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The purpose of this study was to investigate, in terms of constructivist theory, the development of concepts in first-grade children of the fractions one-half and one-fourth. Qualitative methods of observation and interviews as well as quantitative methods from pre- and posttests were used for the study. Informants for the study included six first-grade students. The study was primarily concerned with what children come to know about the fractions one-half and one-fourth and to determine the range of behaviors and actions children use to respond to cognitively based activities involving these fractions.

The major finding from the study suggests that first-grade children's meaning of the fractions one-half and one-fourth can develop from a qualitative unit to a quantitative unit. The findings suggest that children's knowledge of the fraction one-fourth like the fraction one-half has its roots as a qualitative unit. A set of five categories of understanding the fraction one-fourth as a qualitative unit were proposed, reflecting the range of behaviors observed

CHAPTER 1
BACKGROUND FOR THE STUDY

Introduction

The National Council of Teachers of Mathematics (NCTM) published Curriculum and Evaluation Standards for School Mathematics, in 1989. One of its recommended curriculum standards for the kindergarten through fourth grade mathematics is the inclusion of fractions so that students can develop concepts of fractions, gain a sense of fractional numbers, use models to find equivalent fractions, and apply fractions to problem situations (NCTM, 1989).

These goals for the elementary mathematics curriculum are not new, however. They are part of the current curriculum and they are not being met (Steffe & Olive, 1991). Results from national assessments have shown that most students do not have a clear understanding of fractional operations (Behr, Wachsmuth, Post & Lesh, 1984; Carpenter, Lindquist, Matthews & Silver, 1983; Driscoll, 1982; Mullis, Dossey, Owen, & Phillips, 1991; Post, 1981; Suydam, 1984). At the Third International Congress on Mathematics Education, Kieren (cited in Burke, Hiber, & Romberg, 1977) presented a paper that stated that adolescents and adults had not learned the fraction concepts well and that "formal instruction had done relatively little to enrich the experience base that children need to develop" fraction concepts (p. 74). Formal instruction has usually been in the form of direct instruction followed by textbook exercises which is inadequate for developing higher order cognitive skills (Hope & Owens, 1987)

The results of studies by Peck and Jencks (1981) support Kieren's statements. They found that even after three to five years of working with fractions, sixth-grade children had still not developed an adequate conceptual base.

Witherspoon (1993), like the studies by Peck and Jencks, found that students beginning sixth grade were unable to demonstrate that they possessed a meaningful concept of fractions. She concluded that a student able to carry out algorithms may not "understand" fractions.

Hunting and Sharpley suggested in 1988 that students' difficulties with fraction knowledge may be a result from a delay in instruction on fractions. Researchers have shown that children will ascribe whole number meanings to fractions if conceptual bases are not carefully established (Behr, Wachsmuth, Post & Lesh, 1984; Hunting, 1986, Pepper, 1991; Streefland cited in Hunting, 1988). There is some evidence that preschool and primary grade students have some fraction knowledge and can solve fraction-related problems before being taught any appropriate procedures (Clements & Del Campo, 1990; Hunting, 1991; Hunting & Sharpley, 1988, 1991 Polkinghorne, 1935; Suydam & Riedesel, 1971).

It has also been proposed that the difficulties with the concept of fractions are in part due to the instructional procedures and to lack of attention to psychological consequences (Streefland, 1978). There is growing dissatisfaction with the emphasis in fraction instruction on computation rather than on developing the basic ideas needed to learn fractions (Payne, 1984; Post, 1981; Suydam, 1984). Many early researchers who looked at improving fraction learning focused on fraction skills instead of on concepts (Suydam & Dessart, 1980). A number of researchers did investigate basic fraction knowledge and the foundation of learning fractions (Hiebert & Tonnessen, 1978; Novillis, 1976, Post, Behr, & Lesh, 1986; Silver, 1983). Although these researchers asked the question "What constitutes knowledge of fractions?" they did not attempt to answer it from the perspective of the children (Nik Pa, 1989). Children were viewed as passively "absorbing" mathematics structure (Clements & Battista, 1990).

Nik Pa argued in 1989 that knowledge of fractions cannot be transferred ready-made from a teacher to the students. When fractional knowledge is viewed as something that children create, then it is necessary for teachers to be aware of how children create it (Steffe & Olive, 1991) and to present cognitively based activities (Hunting & Sharpley, 1991). Teaching with a constructivist approach is considered effective if activities that enable children to progress at their own pace toward abstract concepts and processes are concrete or semiconcrete (Ohuche, 1990).

The theoretical perspective of constructivism influences the current study. Drawing on Piaget's theories, constructivism is defined as knowledge actively constructed, individually and cooperatively, and "the instruments of construction include cognitive structures that are themselves products of developmental construction" (Noddings, 1990, p. 7).

A number of researchers (Carpenter & Moser, 1984; Hunting & Sharpley, 1988, 1991; Hunting & Davis, 1991; Hunting, 1991) have used a constructivist approach to investigate children's mathematical knowledge. They discovered that children have a rich informal mathematical knowledge prior to formal instruction.

The constructivist approach is appropriate to understand the constructions and representations children make about fractions. This approach has been efficiently applied in formulating specific conceptional models (Leino, 1990). Hunting and Davis (1991) pointed out it is critically important to understand how a child would answer the question "What is a fraction?"

Statement of the Problem

The purpose of this study was to investigate, in terms of constructivist theory, the development of concepts in first-grade children of the fractions one-half and one-fourth. The study was a response to the need to understand the development of knowledge of fractions in children (Hunting & Davis, 1991). The findings from this study have implications for classroom instruction and further research of fraction concepts. When teaching concepts like fractions, teachers must be able to form adequate models of their students' ways of viewing an idea (Confrey, 1990). The insight needed is provided as the findings determine whether young children develop their knowledge of the fraction one-fourth in the same way as for one-half.

Theoretical Base

In contrast to the transmission or absorption view of teaching and learning that most traditional mathematics instruction and curricula are based on (Clements & Battista, 1990), this study's theoretical basis was the constructivist view, which contends that knowledge is actively constructed and "that the instruments of construction include cognitive structures that are either innate or are themselves products of developmental construction" (Noddings, 1990, p. 7).

The assumption is that human beings are knowing subjects, that their behavior is basically intentional, and that they have a great capacity for organizing knowledge (Magoon, 1977). Cobb, Yackel and Wood (1992) contend that students have been constructing their mathematical ways of knowing in the traditional instructional settings. As the teachers were demonstrating the mathematical rules and procedures, students had to make sense of their worlds as they actively interpreted what the teacher said or did. They were only able to interpret to the extent of their current ways of knowing. As Clements and Battista stated "No one true reality exists, only individual interpretations of the world. These interpretations are shaped by experience and social interactions" (1990, p. 34). The central issue is the nature and quality of those constructions. National assessments have shown that students do not have a clear understanding of fractional operations, an indicator of a lack of quality in their constructions. Little is known of the nature of students' constructions of fractional knowledge. Hunting and Davis (1991) assert that it is critically important to learn what understandings children have about fractions and how they reach those understandings.

This study examined what understandings young students have about the fractions one-half and one-fourth as they attempted to solve cognitively based activities. Hunting and Davis (1991) found that children's knowledge of the fraction one-half is initially as a qualitative unit. By qualitative unit, they were referring to the meaning students assigned to language and symbolism of fractions which are literal, context-bound, and imprecise. This study examined children's initial understanding and development of knowledge of the fraction one-fourth and to determined if knowledge of the fraction one-fourth might also have its roots as a qualitative unit.

Methods

The constructivist approach is most appropriate when researchers wish to observe students "as active constructors of their knowledge and rules" (Magoon, 1977, p. 665). The central issue to the researchers is the nature and quality of students' constructions, not whether students are constructing (Cobb, Yackel, & Wood, 1992). Piaget's cognitive constructivism requires researchers to investigate subjects' perceptions, purposes, premises, and ways of working things out to understand their behaviors. Techniques such as clinical interviews and prolonged observations are logical methodologies for Piaget's cognitive constructivism (Cobb & Steffe, 1983; Noddings, 1990; Ohuche, 1990; Thoresen, 1988)

This study principally pursued a qualitative approach, considered more suitable for studying children's cognitive functioning (Davis & Hunting, 1991). The approach included firsthand observations of the students as they experienced the cognitively based activities and the observations of clinical interviews. Quantitative methods in the form of pre- and posttest were also used.

A clinical interview is a conversation between the interviewer and the subject (Davis & Hunting, 1991). The conversation is "centered around a problem or task which has been chosen to give the subject every opportunity to display behavior from which mental mechanisms used in thinking about that task or solving that problem can be inferred" (p. 209). Typically the interviewer poses verbal questions to which the subject responds. Based on the subject's response, the interviewer "asks another question, poses a variation of the task or problem, or in some way sets up a new stimulus situation" (p. 209). This technique allows researchers to gain insight into the thought processes of children. Because of the interviewer's dependence on the subject's response, no two subjects ever receive the exact same interview.

Cobb and Steffe (1983) suggested that the "technique of clinical interviews is ideally suited to the psychological objective of investigating a sequence of steps children take when constructing a mathematical concept" (p. 84). This allows the researcher to trace the mathematical knowledge back to less abstract concepts or operations. A clinical interview gives insights by permitting researchers to understand the meanings students find in mathematical problems (Long & Ben-Hur, 1991). Cobb and Steffe caution, however, that the resulting account does not focus on the critical moments of restructuring. Clinical interviews lack experimental content; children may construct different concepts or take a different set of steps in an instructional setting. Cobb and Steffe recommend that children's constructive processes be observed in their instructional settings and that the researcher observe the children firsthand.

This study provided insight into how first-grade students make sense of their experiences with activities using continuous and discrete units to represent the fractions one-half and one-fourth. The constructivist perspective provides teachers and researchers with a model of how children develop their knowledge of the basic fractions one-half and one-fourth. This has implications for generalizing how children develop their fractional knowledge.

Research Questions

The constructivist perspective that each student has his or her own way of knowing mathematics influenced the research questions.

1. After experiencing cognitively based activities using both continuous and discrete units, how do first-grade students define the fractions one-half and one-fourth during class discussions and interviews?

2. Will the first-grade students' knowledge of the fraction one-fourth have its roots as a qualitative unit?

3. Following experiences with the activities, how does a first-grade student's level of knowing of the fractions one-half relate to knowing the fraction one-fourth in the context of both continuous and discrete quantities?

4. How do first-grade students define, in general, the term fraction?

Significance of the Study

The Professional Standards for Teaching Mathematics (NCTM, 1991) emphasizes the importance of teachers' awareness of how students build their mathematical knowledge. The studies of fraction knowledge done by Hunting (1991) and his colleagues have sought to determine what young children understand about fractions one-half, one-third, and one-fourth and what mental processes exist that may be used as a base for learning fractions. Hunting and his colleagues found only a few children who evidence a mature knowledge of one-half and little understanding of one-third and one-fourth. Their studies have determined, however, that the potential for learning fractions exists in children at a young age. Further studies of the fraction one-half done by Hunting and his colleagues have described categories of knowing one-half as a qualitative unit. My review search indicated that other than one-half, no additional fractions have been studied to determine if some or all understanding of fractions has its roots as a qualitative unit and if categories of meaning for other fractions as a qualitative unit are identifiable.

This study helped to determine if the fraction one-fourth, like the fraction one-half has its root as a qualitative unit. This study will further determine whether a child's category of knowing the fraction one-half is predictive of his category of knowing the fraction one-fourth. This would provide a developmental sequence for knowing of fractions one-half and one-fourth.

To determine the roots of children's knowledge of the fractions one-fourth and one-half, this study needed to observe children who have had no prior formal instruction in fraction concepts. For this reason first-grade students were chosen as subjects. The study determined what relationship first-grade students place between the fractions one-half and one-fourth as well as the roots of fractional knowledge of one-fourth and one-half. This relationship indicated developmental levels of knowledge of fractions which can be useful in producing instruction for children appropriate to their development level.

Definition of terms

The following terms are basic to understanding this study:

Abstract unit refers to the meanings for fractions that are context-free.

A clinical interview is a dialogue or conversation between an interviewer and a subject.

Cognitive constructivism is the belief that knowledge is actively constructed, individually and cooperatively; the method of construction includes cognitive structures that were developmentally constructed (Noddings, 1990).

Cognitive structures are the schemes constructed developmentally from the simple structures that coordinate sensory-motor activity to the complex conceptual structures of mathematics (Skemp, 1987).

A concrete model is the representation of a mathematical idea by a three-dimensional object.

Constructivist approach is an approach developed from knowledge of children's mental processes, and continuous analysis of procedures and representation used by them in learning situations (Bednarz & Janvier, 1988).

Continuous quantities are the representations of a unit or fraction using either length or area.

Cooperative groups are groups with three or four students with mixed abilities working together on a common problem.

Discrete quantities are the representations of a unit or fraction using a set or subset of identical objects.

Fraction refers to a ratio of whole numbers.

Interactions are discussions between students and the teacher or researcher.

Lacking precision refers to a lack of attention by children to whether parts of a whole are equal in size, volume, number, or some other way (Hunting & Davis 1991).

Literal and context-bound refers to student's use of preceding situations or discourse in their definition of a term.

Mathematical interactions are discussions between students as they attempt to solve the activity problems.

A qualitative unit is the meaning a student attributes to the language and symbolism of fractions which are literal, context-bound, and lack precision (Hunting & Davis, 1991).

A quantitative unit is the meaning for fractions that, though largely context-bound, are to some degree transferable across different contexts and attention is given to the precision of the parts (Hunting & Davis, 1991).

Part/whole refers to the interpretation of a fraction that compares a number of congruent divisions to the total number of congruent divisions that would constitute the whole.

Subset/set refers to the interpretation of a fraction that compares the number of elements in a subset of a set to the number of elements in the whole set.

A unit fraction is a fraction whose numerator is one.

Summary

This study investigated the roots of children's knowledge of the fractions one-fourth and one-half. It observed children who have had no prior formal instruction in fraction concepts. For this reason first-grade students were chosen. Another purpose was to determine, in terms of constructivist theory, the development in first-grade students of concepts of the fractions one-half and one-fourth. Since this required detailed analysis of the students during each of the teaching episodes, only 6 children were chosen to be observed and videotaped. The teaching episodes consisted of problem activities. A representative of each group in the class reported to the entire class on the results of the activities. Clinical interviews provided additional insight into the knowledge and development of these 6 children in regard to fraction concepts of one-half and one-fourth. Data collected during the problem activities, interviews, and from group reports and tests provided answers as to how children develop these concepts.

CHAPTER 2
REVIEW OF THE LITERATURE

Introduction

Fraction learning, as one of the major goals of mathematics education in elementary school, is a source of confusion and difficulty for students (Anderson, 1969). It has therefore been the subject of many researchers. Some have investigated adolescents' fraction knowledge after formal instruction; others have studied the fraction knowledge of preschool and elementary school children prior to formal instruction. The following review of the literature covers the research on fraction knowledge of preschool and school age children and includes the studies of the relationship between the constructivist learning theory and the learning of mathematics, particularly of fractions.

Research on Fraction Knowledge in Children

Researchers have found evidence that preschool and primary grade students have some fraction knowledge and can solve fraction problems before being taught any appropriate procedures. Suydam and Riedesel (1971), in their interpretive study of research in elementary school mathematics, reported that studies done by Bjonerud in 1960, Priore in 1957, Wittich in 1942, and Woody in 1931 all found that kindergarten children had some knowledge of fractions.

In 1935, Polkinghorne tested 266 children from kindergarten through third grade. She discovered that children between the ages of 4 and 6 years had some concept of unit fractions, between the ages of 6 and 8 they began to understand proper and improper fractions, and between the ages of 8 and 10 they began to identify fractions. She found that the children knew more about unit fractions when the fractions applied to continuous quantities than when they applied to discrete quantities. Children were also better able to use fractions in comparing two continuous quantities than in comparing discrete quantities. She suggested that the primary school curriculum should include fraction instruction.

Gunderson and Gunderson (1957) interviewed 22 second-grade children to determine their knowledge of fractions. Following lessons on halves, fourths, eighths, thirds, and sixths using paper circles, the children were asked questions about unit fractions, multiple fractions, and using fractions. The study concluded that these children showed an ability to grasp the meaning of fractions and that fraction concepts could be introduced in the second grade.

More recently Peck and Jencks (1981) found that sixth-grade students could conceptualize and make logical predictions about fractions when given physical materials. Peck and Jencks found, however, that less than half the children interviewed were able to demonstrate that they possessed a meaningful concept of fractions and that most of those children who did possess a generalized concept of fractions could not extend their ideas to operations on fractions. Peck and Jencks concluded that even after three to five years of working with fractions, fewer than 10% of the children had an adequate conceptual base. Although children knew to subdivide the whole into the number of parts indicated by the denominator, few knew that the parts had to be of equal size. Results from a study conducted by Burke et al. (1977) found a significant gap between second-grade students' ability to recognize fractional partitioning and their ability to construct fractional partitioning

Clemente and Del Campo (1989) found discrepancies in the understanding of the fractions one-half, one-fourth, and one-third by children in second through fifth grades. Many of the children studied could correctly identify pictures representing these fractions, but when asked to cut a circular disc to obtain one-fourth of the circle or to shade a portion of a given equilateral triangle to represent one-fourth of the triangle, many were unable to respond correctly. Clesents and Del Campo concluded that children need a "classroom environment where children own the mathematics they do" (p. 27).

Steffe and Olive (1991) found that some students had no understanding of a fraction in relation to the fractional whole For example, one 13-year-old student could not find one-fifth of a collection of ten items. To her, one-fifth meant five single items or five parts. Another student, 9 years old, believed that two equivalent fractions were "different objects that were equivalent only because their results produced the same numerosity" (p. 23). Steffe and Olive concluded that learning how students actively construct their fractions and creating a classroom environment that actively involves students are the basic challenges for reform in teaching and learning fractions.

Witherspoon (1993) interviewed seven students the summer before sixth grade on fractions as part of a region and part of a set. She found that only four of the students were able to draw an appropriate representation of one-third of a pie gone. Only one student could correctly represent one-fourth of a set of marbles as white. One student attempted to use a region model and drew four marbles each with a quarter of the marble white. She concluded that experiences with only region models limits students thinking when problems involve other interpretations.

An analysis of 132 interviews with fourth- and fifth-grade students found that children often apply strategies for whole number knowledge to new situations involving fractions (Behr, Wachsmuth, Post, & Lesh, 1984; Post, Behr, & Lesh, 1986). Specifically, it was found that children's ability to order fractions and find the equivalence of fractions was initially influenced by their previous understanding of whole numbers (e.g. 1/5 > 1/4). Children develop or invent strategies for dealing with fraction order and equivalence that have origins in whole number arithmetic.

In a study conducted of 25 low-achieving fourth-grade students (Hiebert, Wearne and Taber, 1991) it was discovered that children do not always make systematic advances in their knowledge of fractions. Their case studies suggest that like the children studied by Behr and his colleagues' studies (1984; Post et al., 1986), these students were influenced by their whole number knowledge in formulating their concepts of fractions. After instruction the students had constructed a partial understanding of fraction concepts. Hiebert et al. found that the movement toward expertise is often filled with progressions and regressions. Students understanding of a concept is advanced but later some or all of that knowledge seems to be forgotten. As Hunting found in his previous work (1983) and in his case study of a fifth-grade student (1986), children can be very resistant to changing their intuitions about fractions.

In a discussion at the annual National Council of Teachers of Mathematics meeting in 1992, Kamii supported the conclusion that children cling to the strategies and algorithms taught in whole number arithmetic. Studying fifth-grade children who had been taught the traditional algorithms, Kamii attempted to "unprogram" these children and have them construct their own algorithms. She found that bright children begin to do their own thinking in two to three weeks, whereas children of lower ability have a difficult time giving up algorithms they had previously learned.

Bezuk (1988, 1989), like Gunderson and Gunderson (1957) and Polkinghorne (1935), has advocated including fractions in the early childhood mathematics curriculum. She has made several recommendations to help teachers remedy or avoid problems in children's understanding of fractions. First, she has suggested using the word names of fractions but delaying the introduction of symbol names until the concept is fully understood. She has also recommended, as have other researchers (Fennema, 1972; Madell, 1982; Peck & Connell, 1991; Post, 1989, Williams & Kamii, 1986), the use of a variety of manipulatives (e.g. rods and pattern blocks) and real-world objects.

In most traditional, formal fraction instruction, the teachers begins by cutting apples or pies in half and then in quarters. Informally, children learn that half means dividing something into two parts and taking one part, but not that the division is into two equal parts (Pothier & Sawada, 1983). Students then progress to shading geometric shapes in halves, quarters and thirds. Once students are competent at writing symbols and shading corresponding shapes, however, computation with fractions begins with little opportunity to manipulate instructional materials (Pepper, 1991). Children therefore learn sets of rules for dealing with fractions instead of developing a conceptual knowledge of them (Baroody & Hume, 1991; Pepper, 1991).

Peck and Connell (1991) have suggested that the use of manipulatives develops mathematics intuition. They have further suggested that physical materials be used not to demonstrate procedures or rules but rather to model the mathematical situations in problems posed to the children. They found that as the fifth-grade students used manipulatives to solve problems they mastered fraction concepts. These concepts allowed students to address problems without discussing in advance a precise method for solving them.

Researchers using discrete quantities (set models) have shown that children of varying ages use sharing as a strategy for establishing basic fraction knowledge. Hunting and Sharpley (1991) found that, although they had no formal instruction, many preschool children demonstrated an ability to share discrete items. Their common sharing strategy is to deal objects (Davis & Pikethly, 1990; Hunting, 1983; Pepper, 1991). When they deal, children distribute discrete objects in a cyclic manner, the same number at each place on each round of the cycle, until no more cycles are possible (Davis & Hunting, 1990). Davis and Hunting questioned whether preschool children would use dealing in situations other than that of a clinical interview. They found a total absence of dealing in the less structured situation; each child grabbed or picked up what he or she considered a fair share. Some of the children were dissatisfied, and discussion and counting ensued to determine fair shares--although those with the larger shares were not inclined to consider numerical evidence. Davis and Hunting concluded that the children either thought that dealing or counting would not help them or that there was no need to establish fair shares.

Pepper in her 1991 study confirmed what Hunting had observed in his 1983 case study: that preschool children used the sharing process to solve a problem of dividing biscuits among dolls. She also found that approximately half the preschool children in her study when asked if each doll had gotten a fair share, had no conception that equality had been achieved. Like the children in Davis and Hunting's study, they seemed unaware that the action of dealing was sufficient to establish a fair share. Second-grade students in Davis and Pitkethly's (1990) study, like the preschool children, used counting to check whether each child had been dealt a fair share and considered it a necessary step in sharing successfully.

Hunting, Davis, and Bigelow (1991) devised the Copy Cat fraction machine, which takes whole number quantities of discrete items as input and, for appropriate quantities, outputs a fraction of the input number. Twenty first-grade and 18 second-grade children were interviewed, with the machine set first to "third" and then to "half." The results were consistent with other studies that have found that one-half was learned earlier and therefore the knowledge of one-half to be more advanced than that of other fractions. The researchers suggested that experience with fraction machines pieced at three different settings would provide children a "potent mental object to which the symbols and language of fractions could become meaningfully integrated" (p. 87).

The second way in which fractions can be modeled is with continuous quantities (area or linear models). Continuous quantities such as area models lend themselves well to fractions (Schultz, 1991). Payne (1984) cited studies that showed children learn the meanings for fractions when they are related to shaded regions. Pothier and Sawada (1983, 1989) found that the ability of children to partition geometric shapes was more extensive than most textbook examples indicated. When the children were asked to verify that the shares were fair, Pothier and Sawada found that they used a variety of methods. Some children used visual estimation to verify that they had produced equal shares. Other children believed that their technique had produced equal shares and that no further verification was necessary. Still others used measurement processes or superimposed the parts. The authors concluded that when children are led to focus on the parts too soon, the whole is made to seem irrelevant. They questioned whether children learn to ignore the whole. Pothier and Sawada have recommended a variety of partitioning activities in early fraction work that may help to alleviate some problems children have with fractions.

Another continuous quantity that can be used for representing fractions is a linear model. One linear model is length Hiebert and Tonnessen (1978) studied 9 children between the ages of five and eight. Each was administered a length task using either halves, thirds, fourths, or fifths. The level of difficulty children experienced corresponded to the number of parts. Hiebert and Tonnessen suggested that the difficulty children had with fourths was that they had failed to use successive dichotomy.

LeFevre (1986) explored fourth-grade students' knowledge about fractions. She found that the number line, another linear model, was difficult for the students to use Bright, Behr, Post, and Wachsmuth in their 1988 study of fourth- and fifth-grade students using fraction representation on a number line, also found that number line instruction was difficult for the students. It requires the use of symbols that may tax the capacity of students to coordinate symbolic and pictorial information. For example, Point A on a number line has no numerical meaning until at least two reference points are labeled. Fraction representation on a number line requires an integration of both visual and symbolic information.

Constructivist Theory in General Terms

Traditional mathematics educators have assumed that children acquire mathematical concepts by internalizing them from the environment (Clements & Battista, 1990). The research and theory of Jean Piaget demonstrates, however, that children acquire mathematical concepts by constructing them from the inside (Kamii, 1985; Kamii & De Vries, 1978). New mathematical knowledge is created as children reflect on their physical and mental actions (Koplowitz, 1979). Ideas become meaningful once they've been integrated into a child's existing structures of knowledge (Clements & Battista, 1990).

Most constructivists in mathematics education agree with Piaget's theory and base on it their interpretation of the cognitive position of constructivism. This position has been described as knowledge actively constructed and the means of these constructions are products of developmental constructions (Narode, 1987; Noddings, 1990). In other words, the higher level constructs are interconnected with constructs and facts below them. Constructivists with this view generally agree that all knowledge is constructed and that the process of construction is continually transforming existing cognitive structures (Leino, 1990; Noddings, 1990; von Glasersfeld, 1990).

When teaching is based on the constructivist theory of learning, two theoretical assumptions emerge (Goldin, 1990). The first is that children, when faced with problematic situations, can develop their own solution methods. Children must be allowed to construct meaning in ways that make sense to them (Wheatley & Clements, 1990). The second assumption is that any knowledge that involves carrying out actions or operations cannot be instilled into children as a ready-made concept but must be actively built up by them. By "actively," constructivists mean that the child plays an active role in interpreting and structuring the new knowledge (Putnam, Lampert, & Peterson, 1989).

Williams and Kamii (1986) have outlined three ways to encourage thinking. The first is to use or create personally meaningful situations in problems posed to children. The second is to provide children with the opportunities to make decisions. The third way to encourage children to think is to provide them with opportunities to exchange viewpoints with their peers. These opportunities can occur in small group work or in class discussions. Children thus develop intellectually and socially as they agree, disagree, or attempt to convince others of their solutions (Clements & Battista, 1990).

Constructivist Theory in Teaching and Learning Mathematics

Many researchers have used a constructivist approach to investigate children's invention of mathematical knowledge. Groen and Resnick (1977) studied preschool children who knew how to count but were unacquainted with arithmetic. With the use of blocks, the children were taught a simple counting algorithm for single-digit addition. After several sessions of practice using the blocks, the children were asked to solve problems without the blocks. Response time for each problem was measured. Groen and Resnick found that the children moved from the counting algorithm they were taught to a counting-on algorithm. They felt it was reasonable to assume that this transition to the new algorithm was an inherent desire for cognitive efficiency.

In a 3-year longitudinal study, Carpenter and Moser (1984) studied first-grade through third-grade children's solutions to simple addition and subtraction word problems. They conducted a series of clinical interviews to study the processes used by the children in solving the word problems and how those processes evolved. The children were interviewed for the first time while they were in first grade, prior to any formal instruction in addition and subtraction. They were interviewed again in their second and their third years, after they had received instruction in addition and subtraction. The authors found in the first interview that 14% of the children had already developed a counting-on strategy for solving addition problems. Nearly half the children first used the counting-on strategy and then counted up to solve missing addend problems. Carpenter and Moser found that children had a rich informal mathematics and suggested that instruction on word problems need not wait until computational skills are mastered.

Bednarz and Janvier (1988) reported the results of another 3-year longitudinal study of a group of children as they advanced from first to third grade. This study developed a constructivist approach to lead children to build a meaningful and efficient symbolism of number. The development of Bednarz and Janvier's approach "required a knowledge of children's thinking (difficulties, conceptions encountered by them), and continuous analysis of procedures and representations used by them in learning situations" (p. 302). The approach they developed helped children "progressively construct a significant and operational system for the representation of number" (p. 329). A follow-up study done three years after the original suggested that the abilities and procedures developed were still present but that conflicts had been generated by the confrontation of the constructivist approach and current teaching methods.

Clements and Battista (1989) investigated the effects of computer programming in Logo on third-grade students' specific geometric conceptualizations. The Logo programming was used as a conceptual framework, not as a direct method of teaching. There was strong evidence that the Logo environment promoted exploration and discussions that had positive effects on the children's geometric conceptualizations. The Logo children were more likely to exhibit signs of transition from a visual level to a descriptive and analytic level of thought.

In 1991 Yackel, Cobb, and Wood reported the results of the 2nd year of a 3-year research project in which they investigated mathematical learning of second-grade students, using educational activities compatible with constructivist learning theory. The instructional strategy involved small-group problem solving followed by whole-class discussion. To permit a detailed analysis of the small-group interaction, eight children, divided into four pairs, were chosen to represent the class. The four pairs were videotaped during their small-group work. Data for this study consisted of the videotapes, field notes, and the children's written work. The researchers found that three types of learning opportunities arise when children work together on mathematics activities:

> (a) the opportunity to use aspects of another's solution activity as prompts in developing one's own solution, (b) the opportunity to reconceptualize a problem for the purpose of analyzing an erroneous solution method, and (c) the opportunity to extend one's own conceptual framework in an attempt to make sense of another's solution activity for the purpose of reaching consensus. (p. 406)

When cognitively based activities designed to be demanding at a variety of different conceptual levels were used, they found the result to be "a cooperative learning environment characterized by active cognitive involvement and elaborated explanations and justifications in the absence of extrinsic rewards" (p.407).

Kamii and Lewis (1993), concluded that children who are allowed to construct their own mathematical knowledge invent more efficient procedures. They found that when children are allowed to invent their own algorithms they are more persistent in finding solutions to problems presented them and develop a better sense of numbers.

Constructivist Views on Teaching and Learning Fractions

Hunting and Sharpley have conducted several studies of preschool children. These studies were guided by questions regarding (a) the behaviors children display when solving partitioning problems using discrete and continuous materials, (b) the mental processes governing these behaviors, and (c) the children's understanding of the fractions half, fourth, and third and of other cognitive structures (Hunting & Sharpley, 1988, 1991). Twenty-two preschool children were interviewed in a preparatory study (1988), and an additional 206 preschool children were interviewed in a later study (1991). Hunting and Sharpley prepared two sets of problems. The first set required each child to share different types of materials among two, three, and four dolls. The second set of problems required each child to represent the fraction one-half and, if successful, to represent the fractions one-third and one-fourth. Few of the children were found to have a deep understanding of fourths and thirds. For many, sharing did not mean that all the material needed to be used, nor that each recipient would be allocated a portion. Hunting and Sharpley described young children's thinking about sharing as "flexible and idiosyncratic" (p. 23). Nevertheless, many of the children used a powerful systematic procedure to accurately find equal fractional units in the problem of partitioning a discrete quantity. Hunting and Sharpley showed that the potential for learning fractions is present at an early age. They recommended that instructional activities that encourage exhaustive distributions and discussions on the merit of using a systematic procedure for making equal units would help develop fraction concepts.

Pothier and Sawada (1983) interviewed 43 children from kindergarten through the first three grades in elementary school. They found that at an early age children observe social situations where things are shared and expressions such as "break in half" and "here's half" are used. Pothier and Sawada showed that children come to understand the term "half" as an operation. Later children come to understand halving as a single division by constructing a line through the middle of a shape. Pothier and Sawada reported however, that at times the two portions were unequal but some children still considered the pieces "fair shares." Pothier and Sawada also observed children making more parts than required or not using all of the material. As Streefland (1978) noted, young children have difficulty grasping the two characteristics of the concept of fraction, namely: "parts should be equivalent" and "the process of subdivision is exhausting; the whole terminates completely into equivalent parts" (p. 55).

Hunting (1991) investigated the origins of children's fraction knowledge in 3-year-olds. He interviewed 21 children and their parents. Like the studies by Hunting and Sharpley, this study found that the children had a successful systematic procedure for solving the sharing task using discrete quantities. An important finding was the way children varied the order in which items were allocated in each cycle of the sharing process and still maintained equality between cycles. Information from the parent interviews suggested that the role of observation and the significance placed on what was observed by the child may be critical in the acquisition of the systematic procedure used by children.

Children's intuitive knowledge of the fraction one-half suggests that the everyday experiences of sharing equally help them to develop the actions and language necessary. Language is intricately connected to the notion of concept (Lerman, 1989). A concept is identified by it uses and gains its meaning from shared social interpretations. The language used by young children would therefore be limited and immanent (Kieren, 1988). Clements and Del Campo in 1990 summarized their research in which they found that preschool children had difficulty performing tasks involving sharing when unfamiliar, formal fraction terminology was used. Hunting and Sharpley (1988, 1991), however, found that when informal language was used, preschool children had little difficulty in sharing. Other studies have found that when the need arose, third-grade students developed verbal expressions such as "one out of five" or "one-fifth" when they observed appropriate situations or when they were required to "take" a certain number of portions from a group (Bergeron & Herscovics cited in Sinclair, 1990; Davydov & Tsvetkovich, 1991).

Collaboration can develop students' ability to verbalize what they know, make comparisons with other students, and negotiate consensuses (Kamii, 1982, 1990; Schifter & Simon, 1992; Tobin, Briscoe, & Holman, 1990). Having to present their own ideas is a "powerful way students come to change or reinforce conceptions" (Brooks & Brooks, 1993 p. 108). It is important to note that this communication sends a set of instructions from which the recipient constructs meaning but that it is not a process of transmitting meaning (Putnam, Lampert, & Peterson, 1989). Children also learn that mathematics is intended to be shared and that success is judged by how well they can express their ideas (Stephens, 1990). The teacher can facilitate the development of this type of mathematical discourse by helping students in their attempts to express their thinking (Kamii & Lewis, 1990) while also encouraging them to reflect on the situation in alternative ways (Wood & Yackel, 1990; Yackel, Cobb, Wood, & Merkel, 1990). This approach fosters children's belief that persisting and figuring out challenging problems is more valuable than a page of correct answers (Yackel, Cobb, Wood, Wheatley, & Merkel, 1990).

Hunting and Davis (1991) narrowed their investigation of preschool children's fraction knowledge to their conception of one-half. They interviewed seventy-five preschool children to discover the procedures the children used to find one-half of discrete and continuous quantities as well as the accuracy of the results. They found that the term "one-half" held different meanings for different children at this age, but that all these meanings were based on a conception of the fraction as a qualitative rather than a quantitative unit. That is, the meanings the children assigned to one-half were literal, context-bound, and imprecise; the children gave little attention to whether the parts of the whole were equal in size.

To explain the observed behaviors of the children in the study, Hunting and Davis proposed eight categories of meaning of one-half, in terms of both the process of findings one-half and the results:

> 1. One half as a multiple sequence of subdivisions. Such actions seem to us to stem either from a lack of awareness of one half as a tare that implies just one cut, or that a distribution process must follow on from the

sequence of subdivisions. . . Multiple products (as a result of the multiple sequence of subdivisions) means that each person will receive multiple pieces according to some method. . . .

2. One half as a single subdivision with either gross or distinct inequality between pieces. We suggest children in this category have had little experience of social imperatives for fair sharing in the sense of equality of quantity at a given point of time. . . .

3. One half as two subdivisions with remainder. In these actions children apparently see no necessity to exhaust the available material, and may never have participated in interactions in which exhaustion is a necessary or useful concept in the context of the use of the term half. . . .

4. One half as a single subdivision with approximate equality and exhaustion . . . . Children in this category may be on the verge of a quantitative notion of half. There is an intention to create equal quantities, but the means to do so may be constrained by nature of the medium . . . A sophisticated behavior involves folding together of endpoints in order to locate the middle . . . .

5. One half as an unequal subdivision of a collection of items with limited potential for equality . . . As with children typical of Category 2 we assume there has been little experience where associations have been made between the use of the term and actions leading to equal quantities. Here a child will be limited in potential to progress until a dealing procedure matures . . .

6. One half as an unequal subdivision of a collection of items, but with potential for equality. The presence of a systematic dealing procedure in the individual's cognition distinguishes this category from the former, but unless there is deliberate action by a teacher or parent to enable the individual to integrate the action of sharing into two equal subcollections with the term one half, progress will not be possible. Children in this category may not be able to check the outcome of their subdivisions using numerical processes such as counting, though they may compare each resulting quantity by direct

> measurement, as for example in comparing the heights of each pile. . . .
> 7. One half as an equal subdivision of a collection of items, as a result of visual check or estimate. A child may respond to term one half appropriately purely by chance, or may use judgments based on gestalt of perceptual field, similar to that required in Categories 3 and 4. Attention to equality is operating, although more sophisticated means such as counting or systematic sharing processes may be unavailable, or not seen to be appropriate. . . .
> 8. One half as an equal subdivision of a collection of items, using a dealing procedure. An individual in this category will confidently share a collection into two equal lots using a systematic dealing procedure. Counting will be used to check whether the outcome is right. Individuals here are on the verge of being able to predict the size of the fractional unit using whole number facts and relationships. . . .
> (pp. 46-49)

These categories of children's thinking of the fraction one-half provide a model needed for the development of the constructivist approach for the present study. Such an approach will allow children to develop and extend their thinking from a qualitative unit to a quantitative unit.

Hunting and Davis (1991) suggest that activities using discrete quantities are crucial if children are to move from a qualitative to a quantitative conception of one-half. The use of discrete quantities provides children opportunities to determine "fair" shares correctly. A critical element of children's understanding of fractions is the recognition that equality (or evenness) of shares is an important characteristic of fractions (Hope & Owens, 1987; Pothier & Sawada, 1983).

Pothier and Sawada (1989) interviewed over 200 children from kindergarten through sixth grade over several years. They identified eight different means of verification or justification children used to explain their judgments of the equality of parts on continuous quantities: (a) visual estimation, using approximation to judge the equality of the parts; (b) confidence in the ability of their technique to produce equal parts; (c) description of the parts with an attempt to compensate for any non-equivalence; (d) measurement with either a formal or an informal measurement process; (e) superimposition of the parts on one another; (f) assumption that parts are equal because they are the same size and shape; (g) reference to the shape of the parts only without necessarily requiring congruency, and (h) looking at the geometry of the whole to determine whether the parts are equal. They suggested that more attention should be given to partitioning continuous quantities during early fraction work. When partitioning of a whole is passed over quickly and the focus is shifted to pre-partitioned shapes the whole becomes irrelevant to the children. Students need time to "manipulate ideas and reflect on them to construct concepts, even after explanations and demonstrations have been given by the teacher" (Battista & Clements, 1990, p. 15).

It is necessary to reconsider the kinds of learning activities in which children are engaged in order to develop their ideas about fractions (Hunting & Davis, 1991). Traditionally the emphasis has been on continuous quantity and measurement experiences, which Hunting and his colleagues (Hunting & Davis, 1991) believe inhibit the transition of knowing fractions as a quantitative unit. Until children are provided with the variety of meanings for fractions and the settings necessary to establish those meanings, little improvement can be expected (Hope & Owens, 1987).

Summary

The constructivist learning theory postulates that knowledge is something learners must construct for and by themselves. According to constructivism, knowledge cannot simply be transmitted or given (Blais, 1988). Researchers have used the constructivist approach to investigate children's invention of mathematical knowledge. Hunting and his colleagues have conducted numerous studies investigating preschool and primary school children's knowledge of fractions. They have suggested that the fraction one-half is the first fraction children understand and that they understand one-half initially as a qualitative unit. They have strongly suggested that children need to have experiences with discrete quantities to move from understanding fractions as qualitative units to understanding them as quantitative units.

The studies of fraction knowledge by Hunting and his colleagues (1991) have found and described categories of meaning for the fraction one-half. No other fraction has been studied, according to my search of the literature, to determine if some or all understanding of fractions is based on a qualitative concept and if categories of meaning for other fractions as qualitative units are identifiable.

The research has shown that preschool and primary grade students have a rich store of informal knowledge about fractions (Clements & Del Campo, 1990; Hunting, 1991; Hunting & Sharpley, 1988, 1991, Polkinghorne, 1935; Suydam & Riedesel, 1971). Children are able to build their meaning of fractions on their informal knowledge thus the potential for learning fractions is present in children at an early age. First-grade students were chosen for this study as the purpose was to investigate, in terms of constructivist theory, children's development of concepts of the fractions one-half and one-fourth. Hunting and Davis (1991) proposed that children move from knowing fractions as qualitative units to knowing fractions as quantitative units. In order for children to progress to knowing fractions as a quantitative unit they suggest that activities using both continuous and discrete quantities are crucial. The use of discrete quantities provides children opportunities to determine "fair" shares correctly. A critical element of children's understanding of fractions is the recognition that equality (or evenness) of shares is an important characteristic of fractions (Hope & Owens, 1987, Pothier & Sawada, 1983). To gain insight into how children development their knowledge of fractions, techniques such as interviews and observations are logical methodologies (Cobb & Steffe, 1983; Noddings, 1990; Thoresen, 1988).

CHAPTER 3
METHODOLOGY

Introduction

The need for a study that looks at how children construct their knowledge of fractions has been documented (Hiebart, Wearne & Taber, 1991; Hunting & Davis, 1991, Hunting & Sharpley, 1988; Hunting & Sharpley, 1991, Kieren cited in Burke, 1977). Until such studies are done, educators will not have the information needed to instill in all students a full understanding of fractions (Hiebart et al., 1991).

This study's purpose was to determine if the fraction one-fourth, like the fraction one-half, has its root as a qualitative unit. The study also attempted to provide a sequence of steps for understanding one-fourth. A primary assumption in methodology was that the knowledge of the students being studied had important consequences for how behavior or actions are interpreted (Magoon, 1977). This requires researchers to investigate the subjects' perceptions, purposes, premises, and ways of working things out if their behaviors are to be understood. Techniques such as interviews and observations are logical methodologies (Cobb & Steffe, 1983; Noddings, 1990; Thomson, 1988).

Entry

In the fall preceding the study the researcher contacted Waterford Elementary School in Orlando with the proposal to conduct a study. The administration was enthusiastic as the school was in the process of developing mathematics curriculum to include constructivist ideas. The administration presented the proposal to the first-grade teachers requesting a volunteer. Ms. Logan asked to be involved in this study. She commented that her weakest curriculum area was mathematics and felt that if she participated in this study it would improve her abilities as a mathematics teacher.

Setting and Subjects

The school chosen for this study is located in a city of approximately 170,000 residents. Opened in the fall of 1991, the school is in middle-to-upper middle socio-economic neighborhoods. The school draws its students from the surrounding neighborhoods and from an area east of the school consisting of very low socio-economic neighborhoods. One school official has stated that some housing in this area is "primitive." Many of the families live in old trailers and the nearest phone is often at the local convenience store. Of the approximately 750 students who attend this school, about 15 percent are from the low socio-economic neighborhoods.

Ms. Logan, the classroom teacher, has been teaching for less than five years. She is a white, middle-class woman in her mid-20s.

At the start of the study there were 26 students in the class. Two weeks into the study an additional student was added to the class. The class consisted of 12 girls and 15 boys between the ages of six and seven. Three of the students were African-American, one student was Asian-American, and a fifth student was Hispanic. Two of the five minority students were in the English as a second language program and did not participate in this study. The percentage of children in this study from low socio-economic neighborhoods was slightly higher than that of the school population. Of the 27 children approximately 22% came from low socio-economic neighborhoods.

All observations were made in Ms. Logan's classroom. She had the students' desks arranged in groups of three to allow for group work. With the absence of the two English as second language students for mathematics instruction there was a total of eight groups, seven of three children each and one group of four. Interviews were conducted in a small room down the hall from Ms. Logan's classroom.

Ms. Logan began each day's mathematics class immediately after the morning routine was completed. During this study she would introduce the problem to the class and then allow the groups to work on solving the problem. The groups usually took 15 to 20 minutes to solve the problem. Ms. Logan would select a representative from each group to report to the class their solution and the strategy they used. The class was encouraged to compare the various group solutions in class discussions during these reports.

Research Procedures

In keeping with the principles of constructivism, the researcher was a participant-observer during the study. The constructivist movement emphasizes direct observation of children as they construct their mathematical knowledge. Direct observation of 6 children permitted a detailed analysis of their construction of fractional knowledge.

Two groups of 3 children each were selected. They were video- and audiotaped. The groups were selected by using a table of random digits.

To minimize the Hawthorne effect two weeks prior to the start of the study the researcher began to observe the participating class. After the first week the researcher, with the full endorsement of the teacher, began to contribute questions and clarifications during the mathematics lessons. In the second week the video and audio equipment was introduced to the classroom to minimize their later effect.

Observational Data

After each teaching episode, the researcher made notes about the problem activity. These notes included children's behaviors observed by the researcher during the activity and comments and suggestions made by the teacher.

Data from Videotapes and Audiotapes

Observational notes written at the end of each teaching episode were supplemented by videotapes and audiotapes, which served as "witnesses" of each teaching episode. Events then could be reconstructed and they provided a record over time of the behaviors of the children.

Cameras and tape recorders were set up to record each of the two groups being studied daily over a three week period. The researcher was present with the cameras to record the interactions between the group members and to record group reports after each teaching episode. The data recorded on the videotapes were later transcribed. Audiotapes were discarded after the videotapes were transcribed.

Quantitative Data from Pre- and Posttests

At the beginning of this study each child was given a pretest of 10 items (see Appendix A). The pretest questions were presented in the same format used on the School Readiness Test (Anderhalter, 1974) and in Dutton and Dutton's (1991) evaluation chapter. The objectives of the test items are, given the fraction (one-half or one-fourth) choose the correct model and, given a fraction shade in the appropriate part of a model.

A posttest of 16 items (see Appendix B) was given after the class has completed the problem sets. The first 10 items of the posttest are based on the objectives used by Novillis in her 1976 study:

> Given a fraction, the student chooses the
> correct model
> Given a model, the student chooses the

> correct fraction (name).
> Given a model, the student selects a model for the same fraction
> Given four models, the student chooses the one that does not suggest the same fraction as the others in the group. (p. 137)

These objectives are applied to the fractions one-half and one-fourth.

The final six items on the posttest were based on the findings of Pothier and Sawada (1983). They described a level of development in children's partitioning processes that they labeled "Evenness, attained when the child attends to the size and shape of the parts and critically examines them to determine if they are equal, or even"(p. 313). These items were included to determine if a child who may not be able to produce equal shares could identify parts as "fair" or "not fair."

Interview Data

The students interviewed included the two female and four male students from the groups video-taped during the class activities. All interviews were conducted six days after the posttest was administered. The interviews, conducted during one day, followed the clinical method and all were videotaped and then later transcribed

The dialogue or conversation between the researcher and the child began with the researcher asking the child to recall an activity involving one-half from the first teaching episode and to demonstrate his or her solution to the problem. The dialogue was centered on problems using continuous and discrete quantities. The researcher posed a verbal problem to which the child responded. The child's response to each situation determined what was introduced next. Since the researcher's questions were dependent on the child's responses, no two children interviews were exactly the same (Hunting, 1983b). The following, however, are questions the researcher may ask:

> How much of the chocolate bar will you get if you share it with me?
> Are the parts equal?
> How can you be sure they are equal?
> What if we shared the chocolate bar among you and three of your friends?
> How much of the chocolate bar would you get?
> Are you sure they are fair shares?
> Given a set of toys, how many toys would you get if we shared them?
> What part of the set do you get to play with?

Data from Book Activity

The final teaching episodes involved the children creating individual books. Each child was given a collection of seven problems involving sharing between themselves and either one or three friends. Three of the problems required each child to share with one character a paper chocolate bar (15.5 cm by 6.7 cm), a long stick of gum (2.5 cm by 16.9 cm), and a piece of string 20 cm long. In the final four problems, each child was asked to share something with three friends. The first three problems involved another chocolate bar, stick of gum, and piece of string respectively. The fourth problem in this group had each child share two cookies (5 cm in diameter) among a group of four. They were encouraged to complete the stories in the problems by sharing the material and writing an ending. The children then bound their completed stories into a books.

Analysis of Data

The data was classified into categories that accounted for the observed behaviors and actions of the students as they responded to the activities in class and during the interviews. The categories proposed by Hunting and Davis (1991) were used by the behaviors and actions of the students as they responded to the activities for the fraction one-half. These categories are: (A) one half as a multiple sequence of subdivisions implying that more than just one cut was made to the material and some process of distribution was used, (B) one half as a single subdivision with either gross or distinct inequality between pieces, (C) one-half as two subdivisions with remainder, (D) one-half as a single subdivision with approximate equality and exhaustion, (E) one-half as an unequal subdivision of a collection of items with limited potential for equality, (F) one-half as an unequal subdivision of a collection of items with potential for equality, (G) one-half as an equal subdivision of a collection of items, as a result of visual check or estimate in which the child may respond to the term "half" appropriately purely by chance or may use judgments based on perception, and (H) one-half as an equal subdivision of a collection of items, using a dealing procedure. Arising from the observations in this study five categories of meaning in the development of students' knowledge of the fraction one-fourth as a qualitative are proposed. These categories are. (1) one-fourth of a continuous quantity as a sequence of more than four subdivisions, (2) one-fourth of a continuous quantity as three subdivisions with remainder, one-fourth of a continuous quantity as three subdivisions with approximate equality and exhaustion, one-fourth as an equal subdivision of a collection of items, as a result of estimation or visual check, and one-fourth as an equal subdivision of a collection of items, through use of a dealing procedure.

These categories were then linked to the following hypotheses. First, that children's knowledge of fractions one-half and one-fourth had its roots as a qualitative unit. Second, that the eight categories of meaning in the development of children's knowledge of the fraction one-half were the same for the fraction one-fourth. Finally, that children defined "fraction" as a unit fraction.

A child's method of verifying the equivalence of their pieces of continuous quantities was classified into categories described by Pothier and Sawada (1989). Of the eight categories, seven dealt mainly with the pieces and only one focused on the whole. These categories are: visual estimation, reference to the technique used, compensatory description of the pieces, measurement process, superimposing the pieces, congruency a requirement, reference to the shape of the pieces, and geometry of the whole informs the partitioning.

For the Fraction Book problems measurements in centimeters were made of the lengths of the resultant pieces. All measurements on pieces that had been cut were made on the edges parallel to the bottom of the page. Measurements made on pieces that had subdivisions marked by pencils were measured on the outside edge of the pencil line. An error score, which expressed the difference between the portion the children assigned to themselves and the remainder as a percentage of the total length of the material was calculated for each "one-half" problem. This score was computed by the formula (a-b)/(a+b), where a is the length of the piece assigned to the child, and b the length of the remainder (Hunting & Davis, 1991).

A standard error score for each "one-fourth" problem, which expressed the error from the true measure of a fourth piece was calculated. The standard error measured the average length the four resultant pieces were from one-fourth the length of the continuous material. This provided a measure to determine students accuracy in creating one-fourth of the continuous materials. The standard error was computed by finding the sum of the squares of the difference between a resultant piece and the true length of a one-fourth piece, divided by four and then found the square root. An error score, which expressed the difference between the portion the children assigned to themselves and the three remaining pieces as a percentage of the total length of the material, was calculated for each "one-fourth" problem. The error score measured how precise the "equivalence" of the pieces created by the students were. This score was computed by summing the absolute values of the difference between length of the piece assigned to the child, and length of each character's piece and divide the sum by the total length of the four pieces. Both the standard error score and the error score provided information to determine the level of precision used by each of the students, an important indicator in determining whether a student is at a qualitative or quantitative level of knowing the fraction one-fourth.

The analysis of each child's pre- and posttest consisted of a t-test. The t-test provided a criterion for comparing the observed difference between the pre- and posttest. Description of the changes in the pre- and posttests are embedded in the discussion of the findings. Results of the quantitative data were compared with the data collected from classroom observations and from analysis of the videotaped sessions of the two groups.

Validity and Reliability

Multiple sources of data were used to insure internal validity (Smith & Glass, 1987). Findings from this study were obtained from observations, videotaping, interviews, and tests. To insure construct validity the pre- and posttests were reviewed by a mathematics education faculty member at State University of New York College at Oneonta who confirmed the test questions did evaluate student's knowledge of fractions one-half and one-fourth.

Inter-observer agreement was estimated by having a reliability observer code videotaped segments after the researcher. Twenty-one of the videotaped segments from the group observations and interviews were observed by the reliability observer. These segments included the student's solutions to the problems as well as their methods of verification of equality as described by Pothier and Sawada (1989). An inter-observer agreement of 90.5% was calculated by dividing the total number of agreements by the total number of segments coded.

The identity of the participating children was protected. Permission to video- and audiotape the children during teaching episodes and interviews had been given by the University of Florida Institutional

Review Board. Informed consent of the parents and students in the classroom were obtained before the study began. The names of the children were changed in the report. Video- and audiotapes were erased as soon as useful data was extracted.

The researcher acknowledges a constructivist perspective. Experiences as a mathematics teacher and graduate student were taken into consideration as a potential source of bias. Every effort was made to record unbiased and accurate data.

CHAPTER 4
RESULTS

Introduction

The purpose of this study was to examine, in terms of constructivist theory, the development of concepts of the fractions one-half and one-fourth by first-grade students. Four research questions guided this study:

1. After experiencing cognitively based activities using both continuous and discrete units, how do first-grade students define the fractions one-half and one-fourth during class discussions and interviews?

2. Will the first-grade students' knowledge of the fraction one-fourth have its roots as a qualitative unit?

3. Following students experiences with the activities, how does a first-grade student's level of knowing of the fraction one-half relate to knowing the fraction one-fourth in the context of both continuous and discrete quantities?

4. How do first-grade students define, in general, the term fraction?

The constructivist methods of data collection used for this study made it possible to observe the students as they actively constructed knowledge of the fractions one-half and one-fourth. The findings of the analysis of the data collected will be discussed under four major headings: (a) categories of knowing the fraction one-half as a qualitative unit, (b) categories of knowing the fraction one-fourth as a qualitative unit, (c) comparison of each individual child's category of knowing fractions, and (d) first-grade students' definition of the term "fraction."

Prior to the discussion of these findings, a brief description of the problems used in the teaching episodes and the structure of these teaching episodes, is appropriate.

Description of Problems and Teaching Episodes

According to constructivist learning theories in mathematics education, children construct their own mathematical knowledge when they can use manipulatives to construct mathematical representations and interact with other individuals (Clements & Del Campo, 1989; Peck & Connell, 1991, Tobin, Briscoe, & Holman, 1990). Each of the problems included the following features: (a) it provided manipulatives for the children to use;

(b) it involved the social activity of sharing; and (c) it was designed to arouse the children's interest and provide motivation in solving it.

Problem 1: Chocolate Bar. The students were shown a large chocolate bar and told that the two teachers, Ms. Logan and Ms. Harder, wanted to eat the chocolate bar. Each group of students was given a paper chocolate bar equivalent in size to the actual bar (19.8 cm by 10.5 cm). The students were asked if they could share the chocolate bar so that each teacher would get a fair share.

Problem 2: Muffin. Each group was given a collection of six paper cupcake liners (2.5 inches in diameter) and a tin. The students were asked to share the liners fairly between the two teachers.

Problem 3. Chocolate Bar. The students were again shown a large chocolate bar and told that this time the two teachers, the principal, and the guidance counsellor wanted to eat the chocolate bar. Each group of students was given a paper chocolate bar equivalent in size to the actual bar. The students were asked if they could share the chocolate bar so that each person would get a fair share.

Problem 4. Muffins. Each group of students was given a collection of 4, 8, or 12 paper cupcake liners and told that four of their classmates wanted to bake muffins. The students were asked if they could share the cupcake liners so that each classmate would get the same amount of muffins.

Problem 5. My Fraction Book. Each child was given a collection of seven problems and a collection of items in a container. They were asked to share these items between themselves and the characters in the problems. After cutting their pieces, they were to glue them onto the story pages and answer questions about the pieces they made. These pages were later compiled into a book. Three of the problems required each child to share with one character. The first problem was the same as Problem 1 except that the paper chocolate bar was smaller (15.5 cm by 6.7 cm). The two remaining problems in this group used different types of continuous quantities. The second problem consisted of a long stick of gum (2.5 cm by 16.9 cm) and the third a piece of string 20 cm long. In the final four problems, each child was asked to share something with three friends. The first three problems involved another chocolate bar, stick of gum, and piece of string respectively. The fourth problem in this group had each child share two cookies (5 cm in diameter) among a group of four.

Social interaction plays an important role in the development of mathematical knowledge in students. To provide more opportunity for interaction in her classroom, Ms. Logan had arranged the 27 student desks into groups of three facing one another. The groups of desks were arranged in a horseshoe, with two groups in the middle and seven groups around them.

Categories of Knowing the Fraction One-Half as a Qualitative Unit

The group reports from each of the groups will be reported first, followed by the observations of the two groups selected for this study. Of interest are the procedures used to arrive at their results. The categories of meaning proposed by Hunting and Davis (1991) for the fraction one-half as a qualitative unit will be used to explain the observed behaviors of the children studied. The categories, which are described in more detail in chapter 2, are: (A) one half as a multiple sequence of subdivisions implying that more than just one cut was made to the material and some process of distribution was used, (B) one half as a single subdivision with either gross or distinct inequality between pieces, (C) one-half as two subdivisions with remainder, (D) one-half as a single subdivision with approximate equality and exhaustion, (E) one-half as an unequal subdivision of a collection of items with limited potential for equality, (F) one-half as an unequal subdivision of a collection of items with potential for equality, (G) one-half as an equal subdivision of a collection of items, as a result of visual check or estimate in which the child may respond to the term "half" appropriately purely by chance or may use judgments based on perception, and (H) one-half as an equal subdivision of a collection of items, using a dealing procedure.

Group Reports

Due to illnesses in the class seven of the eight groups were present for the first teaching episode. The two groups selected for this study were not affected by absences.

Six groups reported using procedures that resulted in a single subdivision of the continuous paper chocolate bar with approximate equality. This procedure is representative of Hunting and Davis's category D. Of the procedures reported by these groups, four used a folding method to find the middle of the bar. One group used a ruler to measure the bar and find the midpoint. The final group used the words "Milk Chocolate" printed on the bar to estimate the bar's middle.

The seventh group reported using a multiple sequence of subdivisions and distributing the subdivisions equally. This procedure is representative of Hunting and Davis's category A.

Each group was asked whether they were sure the shares were equal. Of the four groups that used folding as their strategy, one stated they were not sure if the shares were equal. Two groups appeared to indicate that their method guaranteed equality by simply refolding the bars to show they were equal shares. The final group also appeared to indicate that their method guaranteed equality when they responded that folding placed the line "exactly in the middle (so) it's half." The group using the ruler also responded that their method was correct by saying "we used the centimeter ruler and it looked like it was fair." The group using the words printed on the bar simply answered "yes" without any verification or justification. The group with the multiple sequence of subdivisions justified their strategy by counting the pieces each teacher would get and responding that each was getting a fair share.

In the Muffin problem, three procedures were reported by the eight groups. One-half as an equal subdivision of a collection of items as a result of visual check or estimation, Hunting and Davis's category G, was reported by three of the groups. The groups in this category used the rows in the muffin tin to find each teacher's share instead of using the cupcake liners. Two of the groups volunteered during their report that they had used counting to verify equality of the shares.

Systematic allocation of the cupcake liners, Hunting and Davis's category H, was used by two groups. They each reported that the whole was six cupcake liners and they dealt each teacher three. One group had dealt the shares by ones. The other group had dealt each teacher two initially and then dealt the remaining ones until the material was exhausted. Both of the groups reported that they counted each share to verify that each teacher had a fair share.

The remaining three groups used their whole number knowledge, not represented by Hunting and Davis's categories. They had used their knowledge of doubles to anticipate the outcome of finding one-half of the muffin cups.

During a class discussion following the group reports, Ms. Logan asked the class to compare the chocolate bar and muffin activities. She asked for any similarities or differences between the two activities. Students responded:

Christine: Split that (chocolate bar) in half with you (Ms. Logan) and Ms. Harder and we split these (muffins) in half today.

Steven: Should count to make sure get same amount.

(We) got one section of the chocolate bar but today (we) got three.

Fred: (The) chocolate bar could give you a stomachache so split it with Ms. Harder. If you eat only one cupcake you won't get a stomachache.

Ms. Logan: (to Fred) How many do I have to chose from?

Fred:	Three.
Ms. Logan:	Is it half?
Fred:	Yes.

Procedures Used by the Two Groups Under Observation

The first group under observation consisted of two girls to be called Autumn and Brandy and a boy to be called Carl. The second group consisted of three boys to be called David, Ed, and Fred.

In the first group, Brandy took charge of the paper chocolate bar. With Autumn and Carl looking on, she began to estimate the middle. After several marks were made on the bar, she finally drew a line down the middle. Autumn and Carl began to color in the two sections. After they finished coloring, Brandy folded the paper chocolate bar in half. She unfolded and checked to see if her estimate of the middle was correct. She then began to recolor one section to make it consistent with the fold line. In giving this group's report, Autumn described only that they had used folding to find the middle. She appeared to indicate that the method of folding guaranteed equality of the shares when she refolded the bar in class to confirm that the shares were equal.

In the second group, Ed took charge of the paper chocolate bar and drew two lines lengthwise. Ed explained to David and Fred his strategy. After a discussion, David took charge of the bar and continued to draw lines between Ed's two lines. The boys agreed the top and bottom sections were fair shares. When asked by a teacher what they were going to do with the middle section, David and Ed responded that it would be subdivided into sections and distributed to the teachers. After dividing the middle section into pieces, the boys began to use a systematic dealing procedure to distribute these pieces. When asked why they chose to divide the bar into pieces, David's response indicated that their procedure was influenced by their experience with actual chocolate bars. The chocolate in actual bars is molded in a grid of smaller rectangular pieces. The boys apparently were attempting to recreate those pieces on their paper chocolate bar. When they were asked if each teacher was getting a fair share, they responded by counting the pieces and showed that each teacher got one big piece and eight smaller pieces. Upon further probes as to whether they could show that the shares were fair, David said he could if he had a ruler. David got out his ruler and began to measure the width and length of the paper chocolate bar He noted the measurements but did not change the shares. In the group's report to the class, David described their procedure of dividing the bar into small pieces and dealing them to each teacher.

On the Muffin problem, Brandy divided the pan into two rows, and Autumn began putting the cupcake liners in one row while Brandy began putting liners in the other row. After they filled the tin, they removed and counted the six cupcake liners. They then dealt each teacher three cupcake liners. They reported they "had six and broke it into three and three, each teacher gets three so its fair." They appeared to be using whole number facts of doubling to determine the fair shares.

David took charge of the Muffin problem. He took the six cupcake liners and placed three of them in the tin in a triangular shape (i.e., two liners in the end cups and one in the middle of the row above). He labeled these three for Ms. Logan. He then placed the remaining three in the tin and labeled them for Ms Harder. Ed and Fred disagreed with David's method. They argued that each teacher's muffins had to be in one row. Fred changed the cupcake liners so that one row was Ms. Logan's muffins and the other Ms. Harder's. After the boys changed the pattern several times, they finally decided that no matter what pattern they placed the cupcake liners in, each teacher still got the same amount - three. The boys, like the children in the first group, appeared to use whole number facts of doubling to determine the amount each teacher received. In Ed's report to the class he responded that half means having the same amount for each one.

Categories of Knowing the Fraction One-Fourth as a Qualitative Unit

Findings in this section relate to the view that the first level of knowledge of fractions is as qualitative units - that is, a student attributes to fractions meanings that are literal, context-bound, and lacking precision (Hunting & Davis, 1991).

The group reports from all of the groups for the problems about one-fourth will be reported first, followed by the observations of the two groups selected for this study. Of interest are the procedures used to arrive at their results.

Group Reports

In the Chocolate Bar problem, three procedures were reported by the groups. Four of the groups reported making three subdivisions. Each of these groups folded the bar in half and then folded it in half a second time, getting four pieces with approximate equality.

The remaining four groups reported making a multiple sequence of subdivisions. Three of these groups reported making four cuts, two groups decided to subdivide the left over material and share it four ways, while the third group reported they did not know what to do with the leftover material and decided to save it. The fourth group using multiple subdivisions reported that they had divided the bar in the middle and then counted the marks they had made at the top of the bar. When asked about these marks, they reported that they had recreated the pieces molded on an actual chocolate bar.

Two different strategies of subdividing the cupcake liners in the Muffins problem were described in the group reports. Five of the groups subdivided the collection of cupcake liners by using mental calculations. These mental calculations could have been predictions of the size of the fractional unit by applying whole number facts and relationships.

The remaining groups used a systematic procedure to share their cupcake liners. The groups with 4 cupcake liners all used dealing, while all the groups with 8 or 12 cupcake liners used mental calculations as per above. All the groups were observed by Ms. Logan to have used counting to verify that the shares were fair.

Following group reports from half of the groups, Ms. Logan introduced a new problem to the class. She held up a tin with six cupcakes and asked if it was possible to share these fairly among the four children in our problem. The following are some of the responses given:

Carl: No, (we) would have two (cupcakes) not used if each (child) got one, if each (child) got two (cupcakes) one person doesn't get any.

Brandy: Two (cupcakes) are left over.

David: Each person get one-sixth.

Ms. Logan then gave another problem using two muffins to be shared fairly among four people. A few students responded that it was not possible but gave no explanations. David responded it was possible and gave the following explanation:

David: Cut them in half and you get four pieces. Each person gets one piece.

Procedures Used by the Two Groups Under Observation

In the Chocolate Bar problem, Brandy began solving the problem by estimating and making four subdivisions, which resulted in five sections. Carl and Autumn quickly showed Brandy that they now had five sections. Autumn responded that they had "messed up" and had an extra piece. At first Brandy suggested to the group they could save it for later and the four other pieces would be fair. Ms. Logan asked the group if they felt it was fair to ignore that fifth piece. After discussion between Ms. Logan and the group, Brandy began to divide the fifth piece into four sections by making three subdivisions. Brandy explained to the group that each person would receive one big piece and one small piece. After the group had finished labeling and coloring the sections, Carl took the bar and folded into fourths. He unfolded the bar and checked their sections with the folds. He then straightened the paper without changing any of the shares. The group reported to the class they had "messed up" and had subdivided the extra piece so each person got two pieces.

When David and his group began working on the Chocolate Bar problem, David took charge of the bar. He remembered the measurements from the previous experience with the paper chocolate bar and began to mark off sections that were approximately 2 cm wide. He was unable to find a ruler, so he used his pencil to estimate the sections. He found and marked the middle; then he and Ed counted up the marks David had made. They made four large unequal sections. When asked by Ms. Logan how they could be sure that each person received a fair share, David explained that each person had the same number of pieces. He and Fred began to count the pieces that David had estimated. Fred explained that these were the pieces one sees on an actual chocolate bar. David found that one section had 29 pieces and the other had 30 pieces. Fred suggested that any leftovers would be split into pieces to share. Ed recognized that the shares were not fair, but David resisted changing. After completing their coloring, David folded one section approximately into fourths but went no further.

For the Muffin problem, Autumn, Brandy, and Carl had 12 cupcake liners and a 12-cup tin. Carl immediately suggested that each of the four characters in the story should receive four cupcake liners. Autumn took the tin and said each row had four. She and Brandy placed the cupcake liners into the tin. They then began to label each of the rows with the name of a character in the story. Brandy realized that one of the characters was left out. Autumn suggested changing the columns to rows and she turned the tin 90 degrees. She showed the group that now there were four rows, which would be fair. Brandy removed all the cupcake liners from the tin after writing the names of each of the characters in the bottom of the liners. She counted the liners and dealt the liners by using the name in the bottom of one of the liners. She then verified that the groups of liners were fair by counting. Autumn reported to the class that they had counted the number of cupcake liners they had and had given each person three.

David's group received eight cupcake liners and an eight-cup tin for the Muffins problem. Ed filled the tin with the liners and then removed them. Fred was absent so Ed divide the eight cupcake liners into two groups giving one group to David. For each character in the story, he wrote the name on the bottom of one of the liners. David also wrote names in the bottom of the liners he had. David and Ed began to place the cupcake liners into the tin. They then verified how many muffins each character would get. David had written the same name into two of his liners, so that one character got only one muffin and another received three. When asked if this was fair, Ed responded it was not. David took the extra muffin and wrote the name of the character that had been shorted. When asked if that meant that everyone had the same amount, Ed answered that it did. When asked if they felt it was fair, they answered it was not. Upon probing it became evident that they were unhappy with how the liners were arranged in the tin but agreed that each character did get the same number of muffins.

Throughout the class discussions and during the interviews with the six students, the teachers asked students what one-fourth meant. Prior to the first activity David responded that one-fourth was when you "split in half by four." Following the first activity he responded, "One out of four--one piece of the bar." After the second activity another student, Ben, defined one-fourth as "need to share with four people." After the second activity with one-fourth the following response to the question was given:

Fred: Breaking into fourths--share something with four people who wanted same amount.

Comparison of Each Individual Child's Category of Knowing Fractions

The findings in this section determine how first-grade students define the fractions one-half and one-fourth and how their categories of knowing these fractions relate. These findings relate to the cognitive constructivist view that the knowledge of mathematics children acquire is constructed through their experiences.

Data from Book Activity

Of the seven problems in My Fraction Book, three were about the fraction one-half. The three problems involved sharing with a friend a chocolate bar, a piece of string, and a stick of gum respectively. Table 1 shows the error scores for these three problems. An error score of less than 5% is considered high accuracy (Hunting & Davis, 1991). Three of the students in this study were accurate to within 5% of half in each problem. All of the students were accurate to within 5% of half of the chocolate bar. Five of the students were accurate to within 10% of half of the string, while all of the students that completed the final problem were accurate to within 10% of half of the gum. Low accuracy is an error score greater than 13.5% (Hunting & Davis, 1991).

Table 1: Error Score in Finding One-Half of a Continuous Quantity

Student	Chocolate Bar % Error	String % Error	Gum % Error
Autumn	0.6	2	--[a]
Brandy	0	0	0.6
Carl	0	0	0.6
David	0	10	--[a]
Ed	0	0.6	5
Fred	2	5	7

[a] Student did not complete this activity.

In the chocolate bar, string, and gum problems, the main procedure observed was a single subdivision with approximate equality of the two halves, Hunting and Davis's category D. The students observed folding the material in the middle prior to making a cut had an error score of zero. David was observed using folding for the chocolate bar problem but estimated the middle for the string problem, which resulted in a 10% error. Ed was observed using folding to find the middle of the paper chocolate bar. He too, however, estimated the middle for the string problem and cut before verifying accuracy. After cutting he discovered that the shares were not equal, he then trimmed approximately 2 cm from the longer piece and taped it onto the shorter. Since he overlapped the string in order to tape it, he lost 0.5 cm of string. Fred used estimation on all three of his problems, which resulted in error scores between 2% and 7%, despite discussions of procedures with his group mates.

The last four problems for the book were about the fraction one-fourth. Three problems involved sharing a chocolate bar, a piece of string, and a stick of gum with three friends. Table 2 shows the standard errors for these three problems and the cookie problem. The standard errors for the continuous material indicated the average amount each of the four pieces are from the length of one-fourth of the material. Four of the students were accurate to within 0.78cm of the length of one-fourth of the chocolate bar. Five of the students were accurate to within 1cm of the length of one-fourth of the string. Two students were accurate to within 0.84cm of the length of one-fourth of the gum. The three largest standard errors were a result of the students discarding material.

Table 2: Standard Errors in Finding One-Fourth of Continuous and Discrete Quantities

Student	Chocolate Bar Std. Error	String Std Error	Gum Std. Error	Cookies per Character
Autumn	0.23cm	2.46cm[b]	1.73cm[b]	1/2
Brandy	0.42	0.12	0.25	1/2
Carl	1.06[b]	0.07	--[a]	1/2
David	0.4	0.75	--[a]	1/2
Ed	0.59[b]	0.55	--[a]	1/2
Fred	--[a]	0.50	0.37	1/2

[a]Activity not included in student's book.
[b]Had piece left over

Four of the students who completed the continuous quantity problems (chocolate bar, string, and gum) were observed using one main procedure. They subdivided the material into four approximately equal shares using estimation or folding. Students who used a folding strategy had lowest standard error scores, while those who used estimation were less accurate.

Table 3 shows the error score for each of the three problems using continuous materials. These scores indicate whether the pieces created by the students were equivalent.

Table 3 Error Score in Finding One-Fourth of Continuous Quantity

Student	Chocolate Bar Error	String Error	Gum Error
Autumn	5%	8%	0%
Brandy	9	2	6
Carl	9	1	--[a]
David	11	7	--[a]
Ed	10	a	--[a]
Fred	--[a]	14	7

[a]Activity not included in student's book.

Carl and Ed subdivided continuous quantities into fourths with material left over on one activity while Brandy had material left over on two activities, which resulted in higher standard error scores. Ed trimmed his four pieces in order to approximate equality, his story characters' pieces were within 10% of the length of his piece. Carl divided his paper chocolate bar into five pieces resulting in a standard error of 1.06cm from the length of one-fourth of the chocolate bar, four of his pieces were approximately equal (within 9% of the length of his piece). Of these three students, Autumn had the largest standard errors. Her standard error was 2.46cm from the length of one-fourth of the chocolate bar and 1.73cm from the length of one-fourth of the gum but her characters' pieces were within 8% and 0%, respectively, of the length of her piece. Her explanation was that she "broke it into four with a piece left over."

The final problem involved sharing two cookies with three friends. All the students subdivided the cookies into approximately equal halves and then correctly allocated one-half a cookie to the four children in the story by using a dealing procedure.

Data from Pre- and Posttests

Prior to the beginning of this study, the class was given a pretest. Following the completion of the study a posttest was given. Table 4 shows the scores on the pre- and posttest for each of the six students in the study.

Table 4: Test Scores on Pretest and Posttest

Student	Pretest	Posttest
Autumn	--*	69%
Brandy	50%	56
Carl	40	88
David	90	75
Ed	90	88
Fred	40	94

* Not present that day.

The average change for the five students who complete both the pre- and posttest was 18.2. The t-test was 1.32 with 4 degrees of freedom and the probability was equal to 0.26. The difference between the pre- and posttest was not significant.

Autumn on the posttest missed the questions that required the students to identify the appropriate word for either one-half or one-fourth. She correctly identified the shaded pictures for one-half and one-fourth. She had difficulty in identifying the non-example of one-fourth in item 8 and in recognizing the triangle in item 9 was subdivided into equal portions.

Brandy answered correctly the questions about the fraction one-half on the pretest. She identified one-fourth as one-half on item 2 and shaded three-fourths on item 5 instead of one-fourth. On the posttest, Brandy was not able to identify the correct word for the fractions one-half and one-fourth, but she successfully answered half of the test items about one-fourth. On the final test item, she correctly identified two shapes that had been equally divided as representations of fractions.

On the pretest, Carl correctly answered all the questions about the fraction one-half, but he identified one-fourth as one-half. Carl successfully identified both one-half and one-fourth on the posttest. His only error on the posttest was item 5. He identified one shape that was not divided into equal parts as a representation of fourths and did not recognize that the triangle was subdivided equally.

David correctly answered all the questions about the fraction one-half on the pretest and failed to identify the representation for one-fourth on item 2. On the posttest, he did not recognize two-fourths as being equivalent to one-half and identified non-examples on test item 6. He correctly identified two out of three shapes equally divided as representations of fractions.

Ed answered correctly all but one of the pretest questions. He shaded one part of each shape even though one shape was divided into four sections instead of just two. Ed answered correctly the first eight items on the posttest, but when asked to subdivide a rectangle into two sections, he divided it into 18 pieces. He also chose two shapes that were subdivided into four unequal sections as representations of fractions.

The final student in this study answered correctly two out of three of the questions on the pretest regarding one-half. Fred identified one-fourth as any part shaded in one question and three-fourths shaded as the representations for one-fourth. He shaded three-fourths when asked to shaded one-fourth. Fred scored the highest on the posttest. His only error on the posttest was in his subdivision of a rectangle into fourths, he drew four lines instead of three. Fred was the only student of the six involved in the study to identify all three representations of fractions in the last item of the posttest.

Data from Individual Interviews

Six days after the posttest each of the six students was interviewed individually. Autumn's response to the problem of finding one-half of a paper chocolate bar was to divide the bar into two portions; the estimated middle was 2 mm away from the midpoint. When asked how she knew her portion was half of the bar, she responded:

Autumn: I just looked at it and saw that it looked like half.

Researcher: Can you prove the shares are fair?

Autumn: You could get half and I could get half

Researcher. But how do I know my share is the same as your share?

Autumn took the paper chocolate bar, folded it in half, and unfolded it. She discovered her estimation was off, so she took her pen and redrew the dividing line at the crease and confirmed it was fair. When asked how she would define the fraction one-half, Autumn answered, "It's even, and you could share it with two people."

Another paper chocolate bar was placed in front of her, and she was asked to share it among four people. She immediately picked up her pen and drew a line marking off the first quarter, then continued marking off the other three quarters.

Researcher: What would you call my share of the chocolate bar?

Autumn: First.

Researcher: Out of the whole chocolate bar, how much of it is mine?

Autumn: One out of four.

Researcher: What would you tell someone one-fourth was?

Autumn: Someone gets one and three other people get the three.

Researcher: Does it matter the amount of chocolate each person gets?

Autumn: Yeah.

Researcher: We do not have to worry about it being even?

(Autumn nodded yes.)

Researcher: Yes, we have to worry, or yes, we do not?

Autumn: We have to worry.

Researcher: If we have to worry about it being even, how can we prove that our pieces are even?

Autumn: We could fold it like this.

Autumn then took the bar, folded it on the first line shaded drawn and continued to fold using the first quarter as the measure. When she completed her three folds, the top piece was larger than the others. The researcher asked if the top piece was the same size as the bottom. Autumn, without responding, refolded the top piece to match the bottom piece and attempted without unfolding the bar to correct the size of the pieces. Autumn was then given a piece of string to share among four people. As with the chocolate bar she folded three times with the top piece longer than the other three. When asked if equality was important with the string, she confirmed it was but did not refold the string.

For the first problem using a discrete quantity, Autumn was given a collection of 10 toys and asked to share them among the two of us. She counted the number of toys and then counted out five and gave them to the researcher. She appeared to be using whole number facts of doubling. When she was asked if the two sets were fair, she responded by nodding and saying "five." She appeared to be confident that her method guaranteed equivalent subsets. This problem was followed with a problem of sharing eight toys among four friends. Autumn placed two toys in each group, appearing to use whole number facts. The researcher asked what part of the whole was her share, and Autumn responded, "One out of four." The researcher presented a story problem to Autumn in which she was given a chocolate bar. She was to get one-half of a chocolate bar and her friend Jamie would get one-fourth of the bar. The following dialogue resulted:

Researcher: Who gets more of the bar, you or Jamie?

Autumn: Me.

Researcher: If we give Carl what was left over, how much would Carl get?

Autumn pointed to a quarter piece on the paper chocolate bar she had divided up earlier.

Researcher: What part of the whole is his piece?

Autumn. Um . . . let's see . . . , thirds.

Researcher: How many pieces was the chocolate bar divided into?

Autumn: Fourths.

Researcher: How much was Carl's piece?

Autumn: One out of four.

Brandy's response to the problem of finding one-half of a chocolate bar was to fold the paper bar carefully in half. When asked how she knew her portion was fair, she superimposed the two portions to insure equality. Another paper chocolate bar was placed in front of Brandy, and she was asked to share it with three friends. Brandy carefully folded the bar in half and then folded it in half a second time. When asked why she folded the bar she replied, "All four have the same." Given a piece of string, Brandy folded the string in half and then folded it a second time to find fourths.

For the first problem using discrete quantity, Brandy was given a collection of 10 toys and asked to share them among the two of us. She used a systematic method - she dealt by ones. She was asked if the two sets were fair.

Brandy: Yes, each has five.

Researcher: What part of the group do you have?

Brandy: Five.

Researcher: (After reviewing the chocolate bar)
What part of the group do you have?

Brandy: Got half of the group.

This problem was followed with the problem of sharing eight toys among four friends. Brandy shared the toys by dealing by ones. The researcher asked what one-fourth was. Brandy answered "four people" and rearranged the toys into four sets of one toy each and gave the remaining toys back to the researcher.

The researcher presented to Brandy the story problem in which she was given one-half of a chocolate bar and her friend Jamie would get one-fourth of the bar. The following dialogue resulted:

Researcher: How much are you going to get to eat?

(Brandy pointed to one piece of the paper chocolate bar that had been divided into fourths earlier.)

Researcher: So you are going to get one piece?

Brandy: Two (pointing to two pieces).

Researcher: Why did you change your answer?

Brandy: I get half of the bar.

Researcher: Now, what does Jamie get?

Brandy: One piece.

Researcher: One?

Brandy: One-fourth.

Researcher. How much is left for me to eat?

Brandy: One--one-fourth.

Researcher. So I get to eat one-fourth.

For the chocolate bar problem, Carl estimated the middle of the bar. When the researcher asked him if he could prove the shares were fair, he responded, "This part is as fat as this part. . . . Important that pieces are the same - won't be fair."

Given another paper chocolate bar, Carl was asked to share it with three of his friends. He drew four lines resulting in five sections. When the researcher asked him how much each friend got, Carl realized there was an extra piece. He was asked how he would make just four pieces. Carl responded, "Take each line and move it." He did not change his strategy of drawing four lines.

Given a piece of string, Carl was asked to share it among four people. He subdivided the string into four sections by estimating the ends first and then marking the middle. He responded that his shares were fair without giving any verification other than that his method was correct. The researcher asked him how much of the string was his share, and Carl responded, "One-fourth."

For the first problem using a discrete quantity, Carl was given a collection of 10 toys and asked to share them among the two of us. He counted the whole group and used a systematic method, dealing by ones. When he was asked if the two sets were fair, he simply responded that they were.

This problem was followed with a problem of sharing eight toys among four friends of which Carl was one. Carl shared the toys by dealing. When the researcher asked how many each friend got, and Carl answered, "This." The researcher questioned what "this" was, Carl answered, "One out of four."

The researcher presented Carl with the story problem in which Autumn eats two pieces of a chocolate bar divided into four equal pieces. Carl was asked how much Autumn ate, and he responded, "Half." The story continued with Brandy eating one more piece of the bar Carl was then asked:

Researcher: How much did they leave you?
Carl: One slice.
Researcher: How much of the chocolate bar is it?
Carl: One out of four.

During the story problem Carl came to the realization that he needed to make one less cut than the number of pieces he was attempting to find.

David's response to the problem of finding one-half of a chocolate bar was to fold the paper bar carefully in half. When he was asked what the fraction one-half meant to him, he responded:

David: Half part that you can have if you and me have big bar, break it in half because we could count little pieces to see if same amount. If not can break piece off. There are four in a row, you get two and I get two.

Researcher: Why were you so careful about dividing the bar?

David: If I didn't you won't have the same amount.

Researcher: You said with the bar we had to have the same amount. How can we prove we have the same amount?

David: Can use a centimeter ruler or tape measure to figure it out.

Another paper chocolate bar was placed in front of David, and he was asked to share it with three friends. David carefully folded the bar lengthwise twice.

Researcher: What part of the bar is yours?

David: This one.

Researcher: How much of the bar is that?

David: Four out of one - no, one out of four.

David was then given a piece of string and asked to share it among himself and three friends. He estimated and marked four cuts. He made four pieces approximately the same length, with string left over. The researcher asked him what he would do if all of the string was to be used. David re-marked the string using only three cuts. When asked if he had used all the string and if the shares were fair, he confirmed he had, believing his method to be correct.

For the first problem using a discrete quantity, David was given a collection of 10 toys and asked to share them among the two of us. He counted by twos to 10 and then made two groups of 5 using whole number facts. He nodded yes when asked if the two sets were fair. David used the systematic method of dealing.

Following this problem David was given a collection of eight toys and asked to share them among himself and three friends. David shared the toys dealing by twos.

Researcher: What part of the whole did you get?

David: Two out of eight.

Researcher: If we looked at your part out

of the number of people getting toys,
what would your part be?

David: First.

David was unable to see that his share was one part of four or one-fourth of the collection.

The researcher asked David how he would define one-fourth. David changed the collection of toys to just four toys and holding one of the toys, he said that was one-fourth. The researcher then placed a toy on top of each of David's four groups and asked if it was still one-fourth. David said it was "still part of same group - sticking on each other, some people might not know there's a difference that's when it's one-fourth."

The researcher presented the story problem to David in which he was given one-half of a chocolate bar and his friend Josh would get one-fourth of the bar. David was asked how much Ms. Logan would get of the bar.

David: One piece.

Researcher. What do we call that amount of the whole bar?

David: Four out of one.

Researcher: Four out of one?

David. No, one out of four.

For the problem of finding half of a chocolate bar, Ed began to estimate the midpoint, then stopped and folded the paper bar.

Researcher: Why did you fold the bar?

Ed: So we can get it even, so we can share it in the middle, get a fair share.

Researcher: What do you think one-half means?

Ed: (long silence) That you have something with another person - split it in the middle, split it in half.

Given another paper chocolate bar, Ed was to share among four people. He again used folding to find his answer. He made three folds resulting in eight pieces. Ed counted the number of pieces, then began to refold, this time making only two folds. He traced the second fold marks with a marker before saying that each person got one piece.

Researcher: What part of the whole is your piece?

Ed: Gets this half.

Researcher: Half? How can both of these bars (showing his first chocolate bar alongside this bar) have pieces that are half of the whole?

Ed: (silence)

Researcher: This piece is how much of the four pieces?

Ed: One-half.

Ed was not able to identify that his share was one-fourth of the bar.

This problem was followed with a similar problem using a piece of string. Ed was first asked to share a piece of string among two people. He began by using a piece of paper to measure the string, he then folded the rope in half and marked the middle. When the researcher asked if he was sure that he had found the midpoint, Ed refolded the string to verify. Given another piece of string and asked to share it among four people, however, Ed estimated four small pieces. When asked about the leftover string, Ed insisted that it be saved for the next time. When questioned, he confirmed that his shares were equal.

When given a collection of toys to share among himself and the researcher, Ed initially gave himself five toys, then stopped. He then allotted two toys to each of us. When the researcher encouraged him to continue, Ed dealt the remaining toys among the two of us. He counted each group to verify that we had the same number.

The final problem using a discrete quantity was a collection of eight toys to be shared among four friends. Ed again used dealing, and when asked used a visual check to verify that each person had two toys.

Researcher: What part of the whole is my share?

Ed: Two-fourths.

Researcher: That means I get two groups out of the four groups. Is that fair?

(Ed nodded no.)

Researcher: How much is my share of the four groups?

Ed: Two's.

The researcher then regrouped the eight toys together in front of Ed and asked him to show one-fourth of the collection of toys. Ed gave the researcher back four toys and made four sets of one toy each.

The researcher presented the story problem to Ed: Carl was given one-half of a chocolate bar that was divided into fourths, and Ms. Logan would get one-fourth of the bar. Ed was asked how much of the bar Carl ate.

Ed: Two fourths.

Researcher: Which is how much?

Ed: Half.

Researcher: How much is left over for you to eat?

Ed: One-fourth.

For the chocolate bar problem, Fred estimated the midpoint and divided the paper bar into two pieces.

Researcher: What is one-half?

Fred: Break something that was a whole into two pieces and then if take two things put them together makes the whole.

Researcher: Is there anything special about those two pieces?

Fred: Well, can't take some things and break them.

Researcher: If we can break it, are the pieces special in any way?

Fred: Just break into two pieces.

Researcher: Does the break have to be in the middle of the bar?

Fred: Yes.

Researcher: Does it mean anything that it has to be in the middle of the bar?

Fred: No, just breaks into two.

When Fred was given a paper chocolate bar to divide into fourths, he estimated and drew four lines. The researcher asked if the whole bar needed to be used. Fred responded that the remaining bar could be saved and split later. He used the same strategy with the string problem. When the researcher asked Fred to describe what one-fourth meant he responded,

Fred:	It's like you had four pieces of a chocolate bar and you wanted to eat one you'd call that one-fourth.
Researcher:	Do the pieces have to be equal sizes?
Fred:	If you want to share it with somebody.
Researcher:	What if you are going to share it only with yourself?
Fred:	Have to split it in the same amount of pieces.

For the toys problem about the fraction one-half, Fred counted the total and then counted out five toys for each set. He verified equality of the two sets by counting.

Researcher:	Why did you give me five instead of six?
Fred.	Then I would have four, couldn't split it into even numbers.
Researcher:	Do they have to be even?
Fred:	Yes.

Researcher: Earlier you said "half" is breaking any piece into two pieces. Now you say they have to be even. Do they have to be even?

Fred: Yes, even.

Fred was then given a collection of eight toys to share among four children. He gave each child two toys. When asked what part of the whole his share was he responded, "One out of four."

In the story problem, Fred was told that his friend David ate half of a chocolate bar.

Researcher: If the bar was cut into fourths, how many pieces would David have eaten?

Fred: One-fourth.

Researcher: David ate one-half of the chocolate bar.

Fred: One piece.

Researcher: Is one piece half of the bar?

Fred: Two pieces.

Researcher: Two pieces. Are two pieces one-half of the chocolate bar?

Fred: (nodding) Yeah.

Researcher. You ate one piece, how much is left?

Fred: One piece.

Researcher: Which is how much of the whole bar?

Fred: One out of four.

First-Grade Students' Definition of the Term Fraction

Each of the six students was asked during the interview to say what a fraction is. From the first group, Autumn responded, "First it's a whole, then break it up to fourths, thirds, then half or fifths." When asked by the researcher if those are the only fractions, Autumn responded they were.

Brandy responded, "Halves, thirds, and fourths." The researcher asked if there could be other fractions, and Brandy nodded yes. She took the collection of toys and showed the researcher one-tenth, one-ninth, and so on, continuing to show the unit fractions until she reached one-half.

Carl was asked to describe one-half. His response was, "You cut it in half so it would be the same." When asked to describe what a fraction was, Carl responded, "It's where you divide things in fourths, thirds, half, twelfths."

From the second group, David described a fraction as, "What you've been teaching us. Something you divide stuff--one-half, one-third, one-fourth up to one-thousandth. They have to be equal."

Ed did not give a definition for a fraction. He was able to define one-half after a long silence as, "Something that you have with another person--split it in middle, split it in half." Ed had difficulty in correctly using the language for the fraction one-fourth. He often would call a share one or one-half instead of one-fourth.

The third member of this group, Fred, described a fraction as, "If you had a whole stick of gum and wanted to share it with four people, break the stick of gum evenly so each person had same amount." When the researcher asked if he could give additional examples of fractions, Fred repeated the story problem and then listed unit fractions one-half, one-fourth, and one-eighth. He said, "Fractions can keep going as long as you want them to keep going."

Five of the six students gave a definition of the term "fraction" by listing unit fractions. Four of these students expanded on their definition by giving a description of the term. Although students gave examples of unit fractions not used in any of the activities, their detailed descriptions were bound to activities they had experienced.

These students' descriptions of the fraction one-fourth were also context-bound and literal. In some cases, however, students' meaning of the fraction was to some degree transferrable across different contexts.

CHAPTER 5
CONCLUSIONS AND IMPLICATIONS

Introduction

The purpose of this study was to examine the roots of children's knowledge of the fractions one-half and one-fourth and to determine, in terms of constructivist theory, how first-grade students develop the concepts of these fractions. This study responded to the need to understand how children construct their knowledge of fractions (Hiebert, Wearne, & Taber, 1991; Hunting & Davis, 1991; Hunting & Sharpley, 1988, 1991; Kieren cited in Burke, 1977). Hunting and Davis (1991) stated that it is critically important to learn what understandings children have about fractions and how they reach those understandings.

Research on students' fraction knowledge has assessed the difficulty students have with the concepts of fractions. Research on students' levels of understanding fractions has been limited to the fraction one-half. This study sought to further the understanding of how students come to learn fractions.

When the data was examined, the following three broad categories emerged: meaning the first-grade students place on fractions, categories of knowing the fraction one-fourth, and first-grade students' knowledge of one-fourth. These categories support the major findings of this study which were. (1) first-grade students first understand the fraction one-fourth as a qualitative unit and (2) first-grade students' ways of understanding one-fourth as a qualitative unit fall into five categories.

Meaning First-Grade Students Place on a Fraction

A major finding of this study is that first-grade students' level of knowing the fraction one-fourth, like that of the fraction one-half, begins with a concept of the fraction as a qualitative unit. Three specific findings support the conclusion that knowledge of the fraction one-fourth begins with concepts of the fraction as a qualitative unit.

1. Students' understanding of the fraction one-half began as a qualitative unit.

Students defined the term one-half as sharing between two people. Their descriptions were given in terms of the problems about one-half they had been doing in class. For example, one student during a class discussion described one-half as splitting a chocolate bar and muffins between the two teachers.

In their description of the fraction as a qualitative unit Hunting and Davis' (1991) included a lack of precision as a characteristic feature. Specifically, they found that students gave little attention to whether parts of the whole were equivalent. By contrast, the students in the present study were concerned with equivalence of the parts of the whole. When questioned they agreed to the necessity of equivalence of the two halves and the four quarters during all the activities.

2. The language students used to describe the fraction one-fourth was literal and context-bound.

During the course of the class activities one-fourth was described as sharing among four people or one piece of out of four. In the interviews students used the activities they had done to describe or demonstrate what one-fourth means. For example, Fred's understanding of one-fourth was bound to the chocolate bar activity he had just completed. He described one-fourth as having four pieces and sharing between four individuals.

During the interviews Brandy, David and Ed demonstrated what one-fourth means by constructing four sets of one toy each. They were unable to transfer the meaning of one-fourth to a set of two toys out of a collection of eight. For example, David was shown four groups of two toys each and asked if his group was one-fourth of the collection of toys. He responded that it might be one-fourth if the toys were glued together or if a person could not see that there were two in the group.

3. Students defined the term "fraction" as a unit fraction and, although still largely context-bound, they were to some degree able to transfer across different contexts.

During the interviews some students explained the term "fraction" by listing unit fractions. Their lists were not all limited to the fractions explored during the activities. David described the term as having to divide a quantity equally and then listed the fractions one-half, one-third, one-fourth, and one-thousandth. Fred also described fractions in terms of the activities he had done during the interview and group activities. He also noted, like David, that fractions were not limited to the ones experienced in the study.

Several other students described fractions as dividing a quantity into halves, thirds, fourths, and so on. Like David, these students did not limit their definitions to the fractions they had experienced. Autumn included fifths in her list and Carl included twelfths. Although Brandy listed only those fractions she had recently experienced, she suggested that there could be other fractions.

Five of these students define "fraction" as a unit fraction. Although students gave examples of unit fractions not used in any of the activities, their descriptions were bound to the activities they had experienced. For example, Fred described a fraction repeating the story problem used in the interview and an activity from his fraction book.

At the end of the study five of the students understood the fraction one-fourth as a quantitative unit. Each student could find one-fourth of a group of muffins during class activities, and find one-fourth of a group of toys during the interviews. This would indicate that students saw the unit object one-fourth as flexible and contextually determined.

Categories of Understanding One-Fourth

The second major finding of this study are categories of understanding the fraction one-fourth that reflect the range of behaviors observed. They are:

1. One-fourth of a continuous quantity as a sequence of more than four subdivisions. This sequence of subdivisions means that each person will receive multiple pieces according to some method.

2. One-fourth of a continuous quantity as three subdivisions with remainder. In this action children apparently see no need to exhaust the material.

3. One-fourth of a continuous quantity as four subdivisions with approximate equality and exhaustion. There is an intention to create equal quantities, but the means to do so may not be available.

4. One-fourth as an equal subdivision of a collection of items, as a result of estimation or visual check. A child may respond to the term one-fourth appropriately purely by chance, or may use perceptional judgements.

5. One-fourth as an equal subdivision of a collection of items, through use of a dealing procedure. A child in this category confidently shares a collection into four equal portions using a systematic dealing procedure.

The first category describes the action observed, a sequence of more than four cuts followed by distribution of multiple pieces to each person. This action seemed to stem from a lack of awareness that one-fourth implies three cuts, or from preconceived ideas of the problem which influenced the student's distribution of the material. David and his group's conception of the chocolate bar having sections molded into the chocolate influenced their decisions. Brandy and her group had not realized that in order to make four parts they needed only three cuts. In order to exhaust the material they divided the fifth piece with three cuts. Both groups recognized the necessity for equality and distributed an equal number of pieces to each person.

Students in the second category recognized the necessity of sharing the quantity equally four ways and cut four pieces approximately equal, however they did not exhaust the material. This action, like those of students in the previous category, may have stemmed from lack of awareness that one-fourth implies three cuts. These students choose to ignore the leftover material. For example, Fred during the final interview made four subdivisions that resulted in an extra piece. He reasoned that it could be save for a later time. As Hunting and Sharpley (1991) have suggested, Fred's desire to save the material may be a result of situations at home where quantities were equally distributed but a portion saved for later (e.g., saving dessert). A second example was Ed, he continued to make four subdivisions of the materials until the last problem of his interview, where he surmised that to make four pieces, he needed to make one less cut.

Students in the third category intended to create equal quantities, but the means may not always have been available. A student who makes three subdivisions, checks the resulting portions by direct comparison, then proceeds to trim the pieces in order to equalize the sizes demonstrates an awareness of equality. For example, Autumn, in her fraction book, trimmed material off her four pieces in order to get four approximately equivalent pieces. A sophisticated behavior involves folding the material into fourths, as in the case of David. During the interview, he folded the paper chocolate bar in half and then folded a second time making four equal portions.

Students in the fourth category mentally determined the number of items in each of the four subsets by chance or use of perceptual judgments. Attention to equality was operating, although a more sophisticated strategy such as systematically sharing may be unavailable, or not seen as appropriate. Autumn, David, and Fred are representative of this category. They were able to determine the number of items in each subset without using dealing.

In the final category, students confidently share a collection of items into four subsets using a systematic dealing procedure. Three of the students used a systematic dealing procedure during the interview tasks. Most of students in this category did not appear to feel counting was necessary to verify equality of the subsets. They appeared to believe that the method they used guaranteed equality.

In the present study, the five categories of understanding the fraction one-fourth, similar to those for one-half, appeared to account for the behaviors and actions observed as the first-grade students solved problems during class and the interviews. Unlike Hunting and Davis's categorizations, this list contains no category that includes parts that are not equivalent, as all of the groups as well as the students in the study stated the necessity of equivalence.

First-Grade Students' Knowledge of One-Fourth

After carrying out activities involving the fraction one-fourth, most students were at equivalent categories of understanding the fractions one-half and one-fourth. This finding supports the constructivist idea that knowledge is actively constructed and is interconnected with earlier constructs and facts previously learned.

During the interviews Autumn, Brandy, Carl, and David used the same strategy for solving the problems about the fractions one-half and one-fourth. They subdivided continuous quantities by using approximate equality, Hunting and Davis's category D (one subdivision with approximate equality and exhaustion) for the fraction one-half and category 3 (three subdivisions with approximate equality and exhaustion) for the fraction one-fourth.

For the fraction one-half, Autumn and David subdivided the discrete quantities by counting the total and creating each share using whole number facts indicating that they understand one-half as a quantitative unit, Hunting and colleagues second level of knowledge for fractions. For the fraction one-fourth, they subdivided the discrete quantities by counting the total and created equal shares by using estimation or a visual check, category 4 (an equal subdivision of a collection of items, as a result of estimation or visual check). It was not obvious if these first-grade students were using whole number knowledge, in particular multiplying and dividing by four, to anticipate the outcomes. If these students were using whole number facts then their understanding of the fraction one-fourth would also be as a quantitative unit. Brandy and Carl subdivided the discrete quantities for the fractions one-half and one-fourth using a systematic dealing strategy, Hunting and Davis's category H for the fraction one-half and category 5 (an equal subdivision of a collection of items, through use of a dealing procedure) for the fraction one-fourth.

During the interviews Ed and Fred used the different strategies for solving the problems about the fractions one-half and one-fourth. Both Ed and Fred subdivided continuous quantities into halves using approximate equality, Hunting and Davis's category D (one subdivision with approximate equality and exhaustion), but they were at category 2 (three subdivisions with remainder) for the fraction one-fourth. They did not appear to see the necessity to exhaust the available material, although Ed did subdivided one continuous quantity into approximately equal fourths. Ed appeared to fluctuate between the second and third category for the fraction one-fourth.

Ed and Fred used the same strategies for solving the problems about the fractions one-half and one-fourth with discrete quantities. Ed systematically dealt the discrete quantities to solve problems about both fractions, Hunting and Davis's category H for the fraction one-half and category 5 (an equal subdivision of a collection of items, through use of a dealing procedure) for the fraction one-fourth. For the fraction one-half, Fred like Autumn and David, subdivided the discrete quantities by counting the total. He then created each share using whole number facts suggesting that he understands one-half as a quantitative unit. For the fraction one-fourth, he subdivided the discrete quantities by counting the total. He then created equal shares by using estimation or a visual check, category 4 (an equal subdivision of a collection of items, as a result of estimation or visual check). It was not obvious if he used whole number knowledge, in particular multiplying and dividing by four, to anticipate the outcomes. If he used whole number facts then his understanding of the fraction one-fourth would also be as a quantitative unit.

Findings and Related Research

A constructivist approach to teaching is based on the assumption that when children are faced with problematic situations, they can develop their own solution methods. In a given situation children will construct procedures and language to represent, compare, and operate with fractions (Bell, 1990). In this study all the children observed used the verbal expressions "one out of four" or "one-fourth." David attempted to write one-fourth symbolically in his fraction book using the notation "1 4th." Sinclair (1990) reported research that third-grade students had difficulty with the unambiguous term "half," yet all of the students in this study used the term "one-half." Davydov and Tevetkovich (1991), in the process of testing their system of instruction, found that third-grade students develop, the same type of verbal expressions. Conventional thinking usually places the development of these types of verbal expressions in the upper elementary grades, but it appears that children learn to use these expressions when dealing with situations or when the teacher requires them to describe portions from their groups (Davydov and Tevetkovich, 1991).

Davydov and Tsvetkovich (1991) found that in the process of familiarizing themselves with fractions, students used a whole number to label a portion (as did Brandy in responding "five" for one-half of the group of toys). Some students also stated the total number of pieces from which their subset was taken (as did David in referring to "two out of eight").

In describing what the term "fraction" means, students listed fractions they had not experienced. While most of the students used ordinal numbers they had been exposed to in class, David gave as part of his example "one-thousandth." In all likelihood David had heard this term used at home. As Hunting (1991) found in interviewing preschool children and their parents, there are numerous opportunities in family life for children to develop numerical thinking.

When asked if equivalence of parts was necessary, all the children believed the parts should be equivalent. This belief was indicated by statements that the parts were equal because they were the same size or the same number of pieces. When asked to verify the equality of the parts, the process used by the first-grade students varied. In some cases, students used approximation. Carl used visual estimation to assess equality, explaining, "This part is as fat as this part." Other students verified equality by superimposing the parts, using a folding strategy. Autumn and Brandy when asked to verify equality, refolded the bar and showed the researcher that the parts matched.

Another process was employed by David, who used measurement to verify equality. When asked if the parts were fair, he responded that if he could use a ruler to measure he could verify the parts.

The processes of verification used by these students support the findings of Pothier and Sawada (1989). In their studies of kindergarten through sixth-grade students, they identified eight categories of verification processes. Visual estimation, confidence in ability of their technique, measurement, and superimposing parts were three of them used by these students.

During their interviews Autumn, David, and Fred demonstrated the ability to subdivide a collection of items into fourths, but they had some difficulty in identifying one subset of two items as one-fourth of the whole collection. Although they were able to demonstrate that one-fourth was one equivalent part of four parts, they could not transfer that knowledge to the context of a collection of two items as one-fourth of the whole collection. This finding supports the findings of Novillis (1976) that many of the fourth-, fifth-, and sixth-grade students she studied could associate a fraction such as one-fifth with a set of five objects, one of which was shaded, but could not associate one-fifth with two objects out of a collection of ten.

Alternative Hypotheses

1. The interview process had probes that were too leading or did not follow up on student responses.

Long and Ben-Hur (1991) have suggested that following the presentation of a task to be completed, the interviewer needs to ask additional questions to determine the student's degree of understanding. If a student has difficulty or is unable to find an appropriate approach to a problem, the interviewer suggests a modification of the problem. Interviewers should encourage students to elaborate on their responses.

During the interviews some of the researcher's follow-up questions may be considered leading. One example occurred in the follow-up questions posed to Autumn during the story problem. In response to the question of the size of piece of the chocolate bar given to a friend in the story, Autumn had chosen a chocolate bar she had earlier divided into fourths and pointed to one piece. When asked for an appropriate name for the pieces she had pointed to, Autumn responded, "Thirds." This response indicated that she had focused on the number of characters in the story instead of on the chocolate bar she had chosen. At this point the researcher focused her attention onto the chocolate bar instead of probing her response.

The researcher at times stopped probing after a correct response was given. It may have been possible that the student was not sure of the answer. For example, after Autumn responded, "Thirds," she was again asked for an appropriate name after the researcher focused her attention onto the chocolate bar She responded, "Fourths." The researcher should have followed up on this response to determine if Autumn was sure of her answer.

As part of the interviews, questions such as "Why did you change your answer?" "Can you prove . . .?" "How did you get . . .?" and "Why did you do . . .?" were asked. These questions encouraged elaboration on the actions and the responses students gave and helped to determine students' degree of understanding

2. Dominant personalities in the groups prevented some students in the groups from contributing. It was assumed that each member of the group understood the solution to the activity, but that may not have been the case.

Group interactions provide learning opportunities by encouraging exchange of viewpoints and verbal elaboration (Wood & Yackel, 1990). Group members try to make sense of each other's interpretations and provide one another support. Learning, Wood and Yackel suggest, occurs as members resolve conflicts and attempt to achieve consensus. In the case of the second group observed during this study, one member appeared to be more active in finding solutions. David usually took control of the material and the solution to the problem while Ed often provided the support. Less obvious was the third member's involvement in the group activities. Fred rarely made any suggestions towards a solution but did seem able to follow the solutions proposed by the other members of the group. The difference between David's level of understanding the fraction one-fourth using continuous quantities and the other two boys may be due to his dominance in the group. The difference could also be due to the fact that knowledge actively constructed is very individual. The same experiences can result in different levels of knowledge in different individuals (Cobb, Yackel, & Wood, 1992).

3. The students did not understand the directions to the My Fraction Book activity.

Students were provided with the seven problems that made up this activity. They were instructed to divide the materials among either two or four friends. The resultant pieces were then to be glued onto the appropriate pages and compiled later into a book. Students were given the option of selecting how the problems would be arranged in their book. Final books submitted by the students showed that they understood the directions to the individual problems. Since not all the problems were included in their books, however, it would appear that they did not consider it important to complete all of the problems. One possible reason may have been that students were successful on similar problems and felt it unnecessary to continue. Another possible reason was observed by the researcher. Students solved a problem, cut the pieces of the material, and then set the pieces aside for later. In some cases when the student returned to glue the pieces to the pages, some of the pieces had been lost. Each of the six students in this study completed at least two-thirds of the activities with continuous quantities, a sufficient number for establishing any patterns.

Implications

The findings from this study raises issues for further research. The findings suggested five categories of knowing the fraction one-fourth as a qualitative unit. These students recognized the need for equivalent shares, although they may not have had the means to be precise. It would seem that for these students knowing the fraction one-half as equivalent halves transferred to one-fourth. As these findings are specific to this study, further research using similar qualitative methods may extend the generalizability of this finding. Would students come to know other fractions such as one-third and one-fifth as qualitative units first? Could knowing one-half and one-fourth as quantitative units cause fractions such as one-eighth to be known first as quantitative units? Will their level of knowing one-half ensure equivalence of shares in smaller fractions?

The findings from this study help to answer how children come to know fractions. This understanding may be of interest to elementary teachers as they introduce fractions to their students. Often teachers are insecure in their own abilities with fractions, most have memorized sets of rules and their own concept of fractions has stopped there (Goldin, 1990; Konsin, 1977). In order to improve classroom instruction, teachers need to be aware of how children construct their knowledge and under what contextual situations and social interactions these constructs are facilitated.

Carpenter and his colleagues (1989) found that providing teachers with knowledge on children's thinking does influence their instruction and their students' achievement. With the knowledge of children's levels of knowing one-half proposed by Hunting and Davis (1991) and the categories of knowing one-fourth proposed in this study as well as the importance of both continuous and discrete quantity activities, teachers can better assess and advance their students' knowledge of these fractions. As von Glasersfeld (1990) argued, it is only when teachers have "some inkling of 'where the student is' that ways can be found to guide the student to make an accommodation" (p. 37).

APPENDIX A
PRETEST

Instructions: Read each statement to the class after identifying to the class the block they should be looking at.

1. Circle the square that has one-half shaded black.
2. Circle the glass that is about one-fourth filled.
3. Circle the circle that is one-fourth shaded black.
4. Shade one-half of each of the three shapes.
5. Shade one-fourth of each of the three shapes.
6. Circle all the shapes with one-half shaded.

STUDENT'S ANSWER SHEET

1.

2.

3.

4.

5.

6.

APPENDIX B
POSTTEST

Instructions: Read each statement to the class.

1. Circle the shape that shows one-half shaded.
2. (Hold up picture number 2, a rectangle with one-fourth shaded)

   Circle the word, either HALF or FOURTH, that describes the part of this rectangle which is shaded.
3. (Hold up picture number 3, a rectangle with one-half shaded)

   Look at the rectangle I'm holding. Decide how much is shaded. Now look at the three shapes in box 3. Circle the shape that has the same amount shaded as the rectangle.
4. Circle the shape that shows one-fourth shaded.
5. (Hold up picture number 5, a rectangle with one-half shaded)

   Circle the word, either HALF or FOURTH, that describes the part of this rectangle which is shaded.
6. (Hold up picture number 6, a rectangle with one-fourth shaded)

Look at the rectangle I'm holding. Decide how much is shaded. Now look at the three shapes in box 6. Circle the shape that has the same amount shaded as the rectangle.

7. These two dolls are going to share each of these strange shaped candies. The shaded part is for the doll dressed in the blue outfit. One of the candies does NOT have the same amount of the whole shaded as the other three. Circle the candy that does NOT match.

   Draw line or lines to show how the chocolate bar should be shared so that each doll gets a fair share.

8. Stephanie, Craig, Chris, and David are going to share each of these strange shaped candies. The shaded part is for Craig. One of the candies does NOT have the same amount of the whole shaded. Circle the candy that does NOT match.

   Draw lines to show how the chocolate bar should be cut so that Stephanie, Craig, Chris, and David share equally.

9. Circle the shapes that are divided into EQUAL-SIZE parts.

STUDENT'S ANSWER SHEET

1.

2.

HALF FOURTH

3.

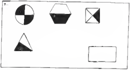
4.

5

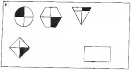
HALF FOURTH

6.

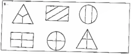
7.



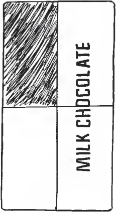
8.



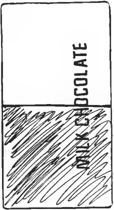
9.



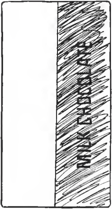
PICTURE 2



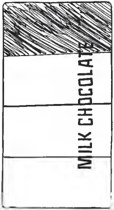
PICTURE 3



PICTURE 5



PICTURE 6



APPENDIX C
STUDENT INTERVIEW GUIDE

Questions will focus on the following areas.

I. Fractions one-half and one-fourth.

Using continuous quantities (area and length) find one-half and one-fourth.

> How much of the chocolate bar will you get if you share it with me?
> How much of the chocolate bar will you get if you share it with three friends?
> Are the parts equal? How can you be sure?
> How much of the string will you get if you are to have one-half?
> How much of the string will you get if you are to have one-fourth?

II. Using discrete quantities find one-half and one-fourth.

> Given a set of ten toys, how many toys would you get if we shared them?
> Given a set of eight toys, how many toys would you get if you are to have one-fourth of them?
> How many children get to play with the toys if each gets one-fourth of this group of toys?

III. Story problems:

> We have one chocolate bar to share. We give Autumn one-half of the whole bar. Then I get one-fourth of the whole bar. How much is left for you?

IV. What does the word "fraction" mean?

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BIOGRAPHICAL SKETCH

Virginia M. Marder was born in Wisconsin and educated in Brevard County in Florida. Brevard County school system was a non-graded experimental school system during her elementary and junior high years. She graduated with her bachelor's degree in mathematics from the University of Florida in 1979.

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In 1985 she taught mathematics at Keystone Heights High School and the following year she moved to Alexandria, Virginia, and taught junior high mathematics.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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